EQUITABLE SHARING OF FINANCIAL AND OTHER ECONOMIC BENEFITS FROM DEEP-SEABED MINING

Supplementary Report Prepared for the Finance Committee of the International Seabed Authority

May 2020

CONTENTS

1. Purpose

This report updates the report on ‘Equitable Sharing of Financial and Other Economic Benefits from Deep-Seabed Mining’ prepared for the Finance Committee of the International Seabed Authority in April 2019 and reported to the ISA Finance Committee in July 2019 (‘the 2019 report’). This supplementary report examines equitable sharing rules or formulae for shares or proportions of the amount to be distributed in any time period for distributions pursuant to Article 140 of the United Nations Convention on the Law of the Sea but not Article 82 distributions. The results are very similar, and any approach for

Article 140 distributions that the Finance Committee settles upon can ultimately be applied to Article 82 distributions with appropriate adjustments.

The 2019 report reviewed alternative approaches to equitable distribution and noted that the concept of common heritage implied that proceeds from activities in the Area would be based, in part, on each country's population as a percentage of the world's total, which would be fully consistent with Aristotle's principle of equity or proportionality.[1] This distribution would then be adjusted through a social distribution weight in such a way as to redistribute income from higher income States Parties to the developing countries referenced in article 140. The report developed a proposed formula based on readily accepted and accessible measures of States Parties' income and populations, adjusted by a social distribution weight to achieve a progressive allocation.[2] This formula is written:

$$S_i = \frac{P_i\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}}{\sum_{i=1}^{N} P_i\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}} \quad ,$$

where S_i denotes the allocated share of States Party i in a time period, $\overline{GNI}$ denotes the average per capita Gross National Income (GNI) of all States Parties, GNI_i denotes the per capita Gross National Income of States Party i, and N denotes the total number of States Parties that receive an allocation (N = 167). It may be noted that replacing $\overline{GNI}$ by the median GNI does not change S_i (because the value appears in both the numerator and denominator of the formula for S_i).

Progressivity was defined to mean that the share of proceeds received by 'low-income' States Parties is higher than the share received by 'higher-income' States Parties. For this purpose, the revealed preferences of States Parties as measured by the scale of assessments agreed by the UN General Assembly was selected as an appropriate metric. In the formula above, the values for η are those revealed by the UN General Assembly through the scale of assessments. The Authority would be able to modify these revealed preferences for η to any value that meets its notion of equity. Lower values of η

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[1] Aristotle's equity principle or proportionality principle states that the goods or services of concern should be divided in proportion to each's claimant's contribution (or claim). Here, the good is homogeneous, divisible, and measured on a cardinal scale in a common metric (US$), and each individual has an equal claim to share Article 140 benefits from deep-seabed mining in the Area due to the status of mineral resources as the common heritage of mankind. This equal claim is adjusted for progressivity in response to requirements of the United Nations Convention on the Law of the Sea (UNCLOS) to redistribute income on a more equitable basis, so that the distribution is not an exact or even one. Instead, the distribution is an even one with unequal entitlements with claimants weighted by social distribution weights. Article 140 of the Convention provides that deep-seabed mining must be carried out for the benefit of mankind as a whole, irrespective of the geographical location of States, whether coastal or landlocked. This implies a joint ownership rationale for equitable sharing. Article 140 also requires the ISA to take into particular consideration the interests and needs of developing States and of peoples who have not attained full independence or other self-governing status, implying an income redistribution rationale as well. Appendix 5 and the original report discuss in greater detail. The same broad principle was applied in a paper prepared by the United Nations in 1971 for the Committee on the Peaceful Uses of the Sea-Bed and Ocean Floor Beyond the Limits of National Jurisdiction. 'Possible Methods and Criteria for the Sharing by the International Community of the Proceeds and Other Benefits Derived from the Exploitation of the Resources of the Area Beyond the Limits of National Jurisdiction' A/AC.138/38, 15 June 1971.

[2] Agreement on such a formula would also imply that the allocation as between claimant States is also a fair division. Allocation is fair division when claimants decide directly through a process of direct bargaining rather than through a third party. Young, P. 1994. Equity: How Groups Divide Goods and Burdens Among Their Members, Princeton University Press, Princeton, pp. 116-117. This is discussed further in Appendix 6.

would reduce the degree of progressivity and larger values of η would strengthen the degree of progressivity.

This supplementary report responds in two ways to the Finance Committee's concerns over the nature of the intra-temporal equitable distribution of allocated shares to States Parties to the United Nations Convention on the Law of the Sea. First, the report develops two new fair and equitable allocation formulae that are added to the original formula and that are *ex ante* based upon principles of equity. Second, the report *ex post* evaluates the allocated shares from each of the three allocation formulae for their intra-temporal equity and contribution to global social welfare using measures of relative inequality and global social welfare. Specifically, the report:

1. Develops two new allocation formula that *ex ante* (prior to the allocation to each States Party) incorporates elements to achieve a more equitable allocation, creating a total of three allocation formulae (the original allocation formula plus two new allocation formula that are in themselves fair and equitable),

2. Evaluates *ex post* (after the allocation to each States Party) the equity and impact upon global social welfare from the allocated share to each States Party from the three alternative allocation formulae using measures of relative inequality and impacts upon global social welfare.

The share allocated to each States Party i, S_i, is a proportion of the total amount to be distributed to all States Parties, where each allocated share ranges between zero and one and the shares sum to one: $0 < S_i < 1$ and $\sum_{i=1}^{167} S_i$. Multiplying S_i by the total amount of royalties or any benefits and costs to be distributed in a time period gives a US$ amount.

2. Ex Ante Equitable Allocation Formulae

All three allocation formulae, predicated upon *ex ante* notions of equity, are based upon Aristotle's equity principle and are weighted by a social distribution weight (arising out of a social welfare function) to incorporate progressivity in terms of income into the equitable distribution:[3]

1. Aristotle's equity principle, represented by each States Party $i's$ share of the population of all States Parties (P_i)
2. Social distribution weights ($\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$), where $\overline{GNI}$ denotes the average per capita Gross National Income (GNI) of all ISA States Parties, GNI_i denotes the Gross National Income of States Party i, and η denotes the elasticity of social marginal utility of income (which is estimated from UN General Assembly annual dues, a form of revealed preference, and the estimated value of $\eta = 1$).[4] Appendix 6 discusses social distribution weights ω_i in greater detail and the original report discusses them in even more detail.

[3] Progressivity is defined to mean that the share of proceeds received by low-income States Parties for Article 140 proceeds (and low-income, landlocked States Parties in the case of Article 82 proceeds) is higher than the share received by higher-income States Parties and high-income, landlocked coastal States Parties, respectively. The reference point is given by mean global per capita income.

[4] This report uses a three-year mean real (constant 2017 prices) 2015-2017 per capita GNI for each States Party, primarily sourced from the World Bank Development Indicators, averaged to smooth out annual and potentially random fluctuations that can impact GNI (e.g. drought, weather, conflict, business cycle, pandemic).

The factors that can be varied when developing the two new, additional formulae include:

1. variables within the allocation formula,
2. functional form of the allocation formula,
3. allocation floor (minimum allocated share to each States Party i, min S_i) and allocation ceiling (maximum share to each States Party i, max S_i), and
4. value of η (which contributes to the degree of progressivity in the social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$).

The three alternative allocation formulae differ by the functional form and whether or not there is an explicit floor and ceiling for the resulting allocated shares (S_i) to each States Party, i.e. min S_i and max S_i. The two variables within the allocation formulae remain the same as the original formula: (1) Aristotle's equity principle represented by share of global population P_i and (2) P_i weighted for progressivity by the social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$.

The impact of values for η greater than $\eta = 1$ (and hence more progressive in an *ex ante* sense) was evaluated in the original report, and while these values $\eta > 1$ impacted the distribution of allocation shares to States Parties, S_i, the relative impact for values of η was less than the change in distribution of S_i required to address concerns raised by the Finance Committee in July 2019. Nonetheless, Appendix 2 *ex post* evaluates the relative equality and impact upon global social welfare of allocated S_i by formal inequality measures for $\eta = 2$ with the original and geometric mean formulae. Appendix 3, as noted, discusses alternative variables in the allocation formulae.

The three alternative formulae presented and evaluated in this report are:

1. Original functional form (original formula)[5]
2. Original formula with floor and ceiling (original formula with minimum and maximum allocated shares S_i)
3. Geometric mean functional form.

All three formulae are related in their basic functional form, since they are versions of a multiplicative functional form called a Cobb-Douglas aggregator function.[6] The three formulae impact the equity of the

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[5] The original formula for States Parties' shares generalizes the 1971 UN paper's Criterion A, paragraph 56. United Nations General Assembly, Committee on the Peaceful Uses of the Sea-Bed and the Ocean Floor Beyond the Limits of National Jurisdiction. 1971. Possible Methods and Criteria for the Sharing buy the International Community of the Proceeds and Other Benefits Derived from the Exploitation of the Resources of the Area Beyond the Limits of National Jurisdiction. Report by the Secretary-General. A/AC.138/38, 15 June 1971.

[6] The numerator in the original formula is multiplicative, because P_i and ω_i are multiplied together. The original formula corresponds to a Cobb-Douglas aggregator function of: (1) Aristotle's equity principle represented by P_i and (2) progressivity represented by ω_i, with exponents of one for each of these two variables in the numerator for each States Party. Appendix 7 discusses the nature of the Cobb-Douglas and other potential aggregator functions in greater detail. Appendices 3 and 8 discuss the aggregation issue if additional criteria C_{ij} are added. Appendix 8 discusses how relative weights can be developed for the additional criteria C_{ij} (through, for example, voting or points systems or choice experiments). These weights could conceivably replace the weights of the three alternative formulae developed in this report, in which the original formula and original with ceiling and floor have

distribution of the allocated shares S_i as measured by the distribution's overall skewness, minimum and maximum values of the allocated shares S_i , and the equity of the distribution as measured by formal measures of relative inequality, several of which also measure the impact upon global social welfare (in terms of a social welfare function), and that have been developed in the economics literature on income inequality (briefly summarized in Appendix 4).

The original formula is written:

$$S_i = \frac{P_i\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}}{\sum_{i=1}^{N} P_i\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}} \ ,$$

where S_i denotes the allocated share of States Party i in a time period, $\overline{GNI}$ denotes the average per capita Gross National Income (GNI) of all States Parties, GNI_i denotes the per capita Gross National Income of States Party i, and N denotes the total number of States Parties that receive an allocation (N = 167). Replacing $\overline{GNI}$ by the median GNI does not change S_i (because the value appears in both the numerator and denominator of the formula for S_i).

The share of total population of each States Parties, P_i, adjusted by the social distribution weight, $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$, yields greater benefits to those beneficiaries with a larger share of total population P_i (and thereby satisfying Aristotle's Equity Principle) and populations with per capita GNI less than the mean per capital GNI through larger social distribution weights ω_i (and thereby creating a more progressive allocation as required by UNCLOS).

The original allocation formula with a floor and ceiling for the allocated shares S_i ensures a minimum allocated share for each States Party, notably States Parties with small populations (and hence small values of share of total population P_i) and ensures a maximum allocated share for each States Party. Ensuring a maximum allocated share S_i precludes any individual States Party i from receiving what could be viewed by some States Parties as a disproportionate share.

Put another way, Aristotle's Equity Principle, as modified by the UNCLOS progressivity requirement, applied in relation to resources having the status of the Common Heritage of Mankind can be applied to individual persons, so that each person has an equal claim, or this principle can be applied to individual States (this approach was considered but not implemented in the three formulae, but is discussed below in Appendix 2). Nonetheless, a floor and ceiling for S_i can be thought of creating a hybrid of Aristotle's Equity Principle applied to individual persons and individual States Parties. The floor or minimum allocated share, i.e. min S_i, is determined from the revealed preference floor from the annual UN General Assembly minimum amount paid by States Parties: S_i = 0.00001, i.e. S_i = 0.001%. The ceiling or maximum allocated share, i.e. max S_i, is determined from the revealed preference ceiling of the International Seabed Authority maximum amount paid by States Parties for their annual contributions to the overall budget: S_i = 0.1631, i.e. S_i = 16.31%.[7] Using a floor and ceiling allocated share, along with

equal weights of one and the geometric mean formula has relative weights (exponents) of one-half (since there are two variables to be aggregated, P_i and w_i.

[7] S_i was increased to slightly more than the floor amount of S_i = 0.00001 for all States Parties for which initially S_i < 0.00001. Due to additional redistribution of shares, the actual floor amount became S_i = 0.0000112. S_i = 0.1631 was allocated to the single large States Party with $S_i > 0.1631$. S_i in excess of S_i = 0.1631 was redistributed from

the social welfare weight ω_i, create a less skewed distribution and more equitable distribution (as evaluated *ex post* by formal inequality measures) for the allocated shares and contribute toward equity with a stability property called "no justifiable envy".[8] Note: For the Authority, the ceiling assessment rate is 22 per cent, and the floor rate is 0.01 per cent. However, since no State Party currently reaches the ceiling rate, the actual ceiling for the Authority from 2021 will be 16.31 per cent. For the purposes of the illustrative analysis in this report, the ceiling rate of 16.31 per cent is used.

The geometric mean functional form for the allocation formula is written:

$$S_i \frac{\left[\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} * P_i\right]^{\frac{1}{2}}}{\sum_{i=1}^{N}\left[\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} * P_i\right]^{\frac{1}{2}}} = \frac{\left[\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}\right]^{\frac{1}{2}} * \; P_i^{\frac{1}{2}}}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=\frac{1}{2}} * P_i^{\frac{1}{2}}}$$

The difference between the geometric mean formula and the original formula is that each term P_i and ω_i is raised to the power $\frac{1}{2}$ rather than 1. Appendix 7 discusses functional form in greater detail.

3. Ex Post **Evaluation of Equity of the Allocation Shares from Each Formula**

Equity of allocated shares S_i for each of the three allocation formulae is evaluated by *ex post* analysis using empirical and formal measures of inequality and global social welfare. These measures include:

1. Gini coefficient, Lorenz curves, and Pen's Parade of Dwarves,
2. Atkinson inequality measures,
3. Generalized entropy measures.

The Gini coefficient, Lorenz curve, and Pen's Parade primarily assess relative inequality per se, but the Gini coefficient and Lorenz curve can be related to social welfare functions under certain conditions. Two measures assess both relative inequality and global social welfare (as determined from a social welfare function): (1) Atkinson inequality measures and (2) Generalized Entropy measures. Appendix 5 explains in greater detail the Atkinson and Generalized Entropy measures of relative inequality and social welfare.

4. Empirical Results

4.1. Summary of Empirical Results

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this States Party with $S_i > 0.1631$ to all other States Parties, including those at the floor of S_i = 0.00001. The redistribution was according to the original formula for all States Parties except the States Party with S_i = 0.1631 (which was held constant) and those with S_i < 0.00001 which now started from a base of S_i = 0.00001. Thus, all States Parties except the one with S_i = 0.1631 received a larger share using the recalculated original formula (and starting from S_i = 0.00001 for relevant States Parties).

[8] Equitable sharing has justifiable envy if a States Party would prefer another allocation to that which it receives when a States Party of higher income receives a larger allocation of proceeds.

The primary empirical results can be summarized as follows (with $\eta = 1$, where Appendix 2 gives results for $\eta = 2$):

1. The allocated shares S_i from the geometric mean allocation formula have the greatest global social welfare and give the most equitable distribution (the lowest relative inequality) of the three formulae when considering all shares for all States (globally).
    a. Thus, the ranking of the three formulae in terms of equitable distribution and global social welfare is from highest to lowest: geometric mean > original with floor and ceiling > original.
2. The geometric mean allocation formula is relatively most equitable and has highest social welfare when per capita GNI < mean per capita GNI, i.e. when $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} > 1$, as determined by the Gini coefficient, Lorenz curve, and Atkinson and Generalized measures of relative inequality.
    a. Thus, the ranking of the three formulae in terms of equitable distribution and global social welfare when $\omega_i > 1$ is from highest to lowest: geometric mean > original with floor and ceiling > original.
3. The geometric mean formula has a minimum allocated share S_i ($minS_i = 0.0000272$) that exceeds the minimum S_i of the original ($minS_i = 3.77e - 08 = 0.0000000377$) and exceeds the original with floor (minimum, $minS_i = 0.0000112$) and ceiling (maximum) formulae.
    a. Thus, the minimum shares $minS_i$ for the three formulae ranked from largest to smallest is: geometric mean > original with floor and ceiling > original.
4. The geometric mean formula has a maximum allocated share S_i ($maxS_i = 0.0778$) that is less than the maximum S_i of the original ($maxS_i = 0.3078$) and the original with floor (minimum) and ceiling (maximum, $maxS_i = 0.1631$) formulae.
    a. Thus, the largest shares $maxS_i$ for the three formulae ranked from smallest to largest is: geometric mean > original with floor and ceiling > original.
5. The geometric mean formula has more allocated shares S_i that are "bunched together" in the "middle" of the distribution and is less skewed than the original and the original with floor (minimum S_i) and ceiling (maximum) formulae.
    a. Thus, the skewness for the three formulae ranked from least skewed to most skewed is: geometric mean (3.92) > original with floor and ceiling (5.82) > original (10.11).
6. The ranking of the original and geometric mean formulae for values of $\eta = 1$ and $\eta = 2$ in terms of most equitable and highest social welfare from highest to lowest is: geometric mean $\eta = 1$ > geometric mean $\eta = 2$ > original with floor and ceiling $\eta = 1$ > original $\eta = 1$ > original $\eta = 2$. The Atkinson and Generalized Entropy (Theil) Inequality Measures, Gini Coefficient, and Lorenz Curve results reinforce the conclusions of the histograms and kernel density estimators that raising η from $\eta = 1$ to $\eta = 2$ paradoxically creates more losers than gainers and decreases equity and global social welfare when reallocating proportions or shares of a fixed amount on the basis of η. A limited number of States Parties enjoy exceptionally large gains in allocated shares regardless of the formula.
7. The equity of distribution to ISA regions depends upon heterogeneity of each region's States Parties by population share P_i and to a lesser extent the magnitude of each States Party i's social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$. The ranking of equitable distribution by ISA region from the most to least equitable distribution (where relative equity is determined by the Atkinson and Generalized Entropy measures) is:
    1. Eastern European Group

2. Western European and Other Groups
3. Africa
4. Latin American and Caribbean States Parties
5. Asia Pacific Group

8. The same ranking of the distribution for social welfare is found as with the ranking of relative inequality, i.e. the EEG group receives highest social welfare relative to others, WEOG next most, etc.
9. Changing the distribution formula is the best way to alter the equitable distribution of allocated shares S_i. Paradoxically, raising the progressivity parameter η, the elasticity of the social marginal utility of income from $\eta = 1$ to $\eta = 2$ lowers rather than raises the distribution of allocated shares' equity and social welfare. Raising the value of η creates proportionately more losers than gainers and a limited number of gainers enjoy considerable gains in allocated share S_i.
10. Although not reported here, there is a very similar and consistent pattern for both Article 140 and Article 82 distributions.
11. A statistical (generalized linear model regression) analysis shows that share of population P_i has several orders of magnitude greater impact upon S_i than does the social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$ for all formulae.
    a. Even when excluding P_i from the formula for S_i, so that the formula depends only upon $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$, the statistical analysis gave the same result (that P_i's impact upon S_i is orders of magnitude larger than ω_i). Similarly, the correlation coefficient between P_i and S_i is substantially larger than the correlation coefficient between ω_i and S_i (both are always statistically significant).

4.2. Empirical Results in Detail

The balance of this discussion now examines the relative inequality and the impact upon global social welfare of the distribution of the allocated shares S_i from the three different allocation formulae in terms of an *ex post* analysis using measures of relative inequality and impact upon global social welfare.

Table 1 reports summary statistics for the three allocation formulae.

Tables A.1.1.-A.1.3. in Appendix 1 provide detailed summary statistics by percentile of recipient for each of the three allocation formulae.

Evaluating the distribution of the allocated shares when $\eta = 2$ rather than $\eta = 1$ assesses the sensitivity of the distribution to a higher value of the progressivity parameter η. Appendix 2 provides more detail upon the distribution of the allocated shares for $\eta = 2$,

Table 1. Summary Statistics of Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae

Type of Allocated Shares S_i	Mean $\eta = 1$	Skewness $\eta = 1$	Skewness $\eta = 2$	Minimum Share $\eta = 1$	Minimum Share $\eta = 2$	Maximum Share $\eta = 1$	Maximum Share $\eta = 2$	More or Less Compact with Larger η?
Original	0.0060	10.11	7.82	3.77e-08	3.72e-10	0.3078	0.2833	More
Geometric Mean	0.0060	3.92	4.11	2.72e-05	3.44e-06	0.0778	0.0948	More
Original with Floor (0.00001) and Ceiling (0.1631)	0.0060	5.82		0.0000112		0.1631		

Note: A blank cell for the original formula with a floor and ceiling arises since the allocated share S_i was not calculated for $\eta = 2$. Mean share values (column 2) are arithmetic means.

The geometric mean formula (with $\eta = 1$) has a minimum allocated share S_i ($minS_i = 0.0000272$) that exceeds the minimum S_i of the original ($minS_i = 3.77e - 08 = 0.0000000377$) and exceeds the original with floor (minimum, $minS_i = 0.0000112$) and ceiling (maximum) formulae. Thus, the minimum shares $minS_i$ for the three formulae ranked from largest to smallest is: geometric mean > original with floor and ceiling > original.

The geometric mean formula (with $\eta = 1$) has maximum allocated share S_i ($maxS_i = 0.0778$) that is less than the maximum S_i of the original ($maxS_i = 0.3078$) and the original with floor (minimum) and ceiling (maximum, $maxS_i = 0.1631$) formulae. Thus, the largest shares $maxS_i$ for the three formulae ranked from smallest to largest is: geometric mean > original with floor and ceiling > original.

Larger skewness values correspond to a more skewed distribution of the allocated shares S_i, notably a longer tail for larger values. Conversely, a less skewed distribution is more compact than a more skewed distribution. Thus, the skewness for the three formulae (with $\eta = 1$) ranked from least skewed to most skewed is: geometric mean (3.92) > original with floor and ceiling (5.82) > original (10.11).

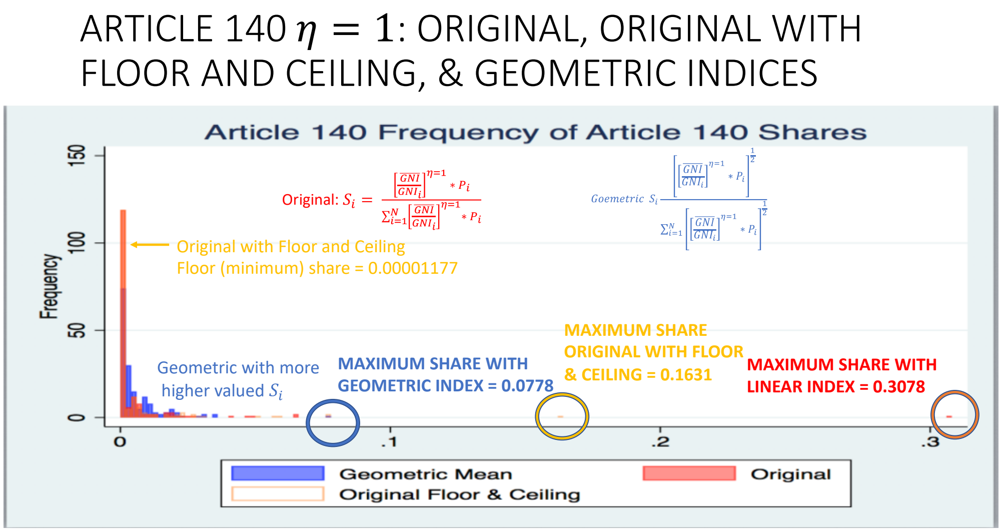
The distribution of the allocated shares S_i for the three formulae (with $\eta = 1$) can be visually displayed by the histogram in Figure 1. In the figure, the original formula is depicted by red, the original with floor and ceiling is depicted by orange, and the geometric mean formula is depicted by blue. Appendix 1 has a histogram and kernel density estimation[9] (essentially a smoothed histogram) for each individual formula.

Figure 1 clearly shows that the order of most skewed to least skewed distribution of allocated shares S_i is original (red) > original with floor and ceiling (orange) > geometric mean (blue). The original formula (red) has the lowest shares (S_i = 3.77e-08 or 0.0000000377) of the three formulae, although that cannot

[9] Kernal density estimation helps visualize the "shape" of data, as a type of continuous replacement for the discrete histogram. In statistics, kernel density estimation is a non-parametric way to estimate the probability density function of a random variable. Kernal density estimation is a fundamental data smoothing approach.

be seen from the histogram due to the degree of resolution. The original with floor and ceiling formula (orange) has the highest frequency of minimum shares at the floor level, followed by the geometric mean formula (blue), in turn followed by the original formula (red). The original formula (red) has a maximum value of 0.3078 which exceeds the maximum value of the original with floor and ceiling formula (orange) of 0.19, which in turn exceeds the maximum value of the geometric mean formula (blue) OF 0.0778. The original formula (red) and the original with floor and ceiling formula (orange) both have a higher frequency of high-valued allocated shares than the geometric mean formula (blue).

Figure 1. Histogram of Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae



Kernal density estimators, which are essentially smoothed histograms, gives essentially the same results as for the histograms.

Figure 2. Kernal Density of Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae

ARTICLE 140: ORIGINAL, ORIGINAL WITH FLOOR AND CEILING, & GEOMETRIC INDICES, ! $= 1$

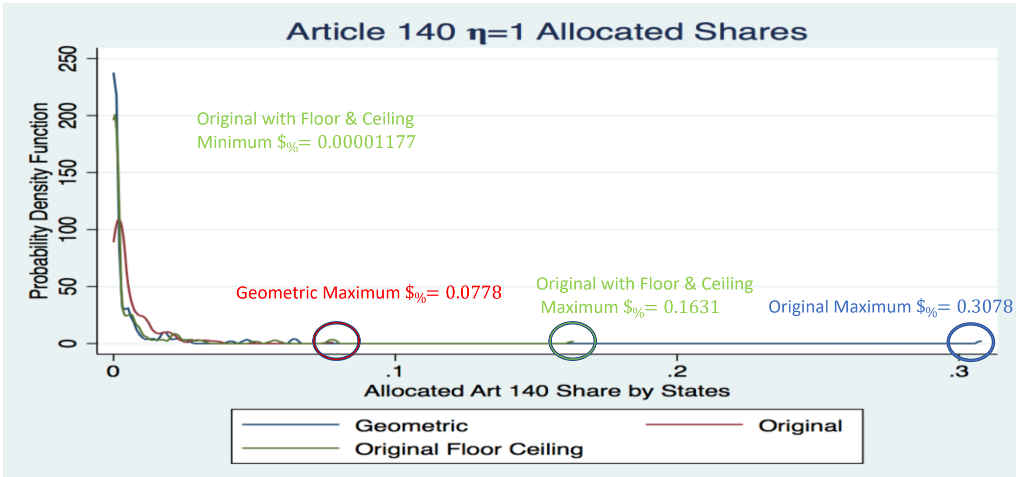
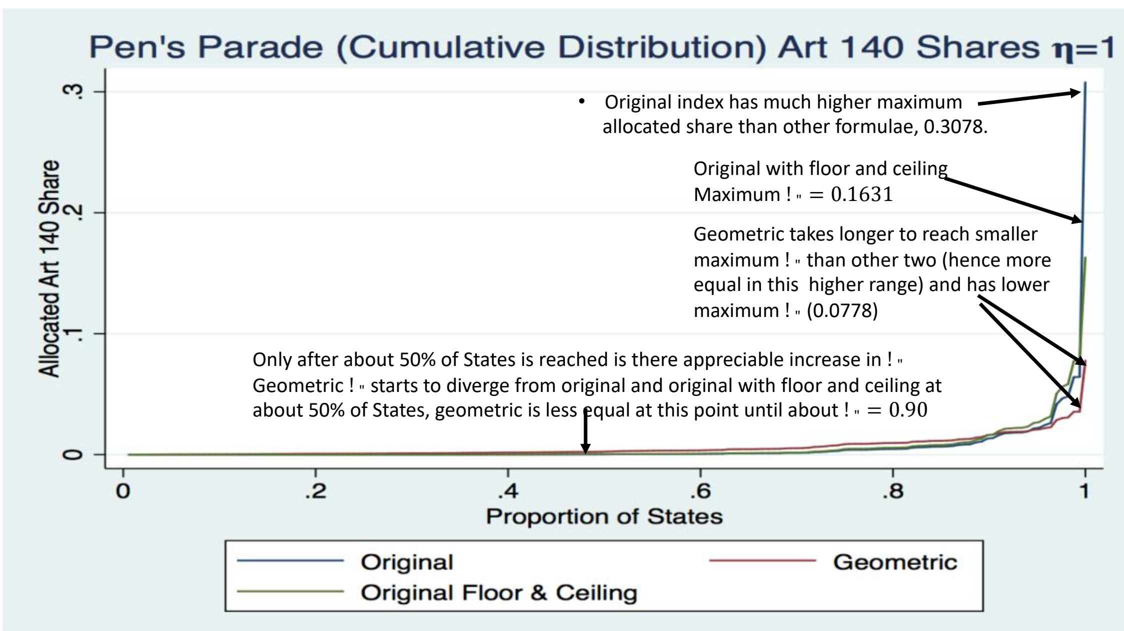


Figure 3. Pen's Parade of Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae[10]

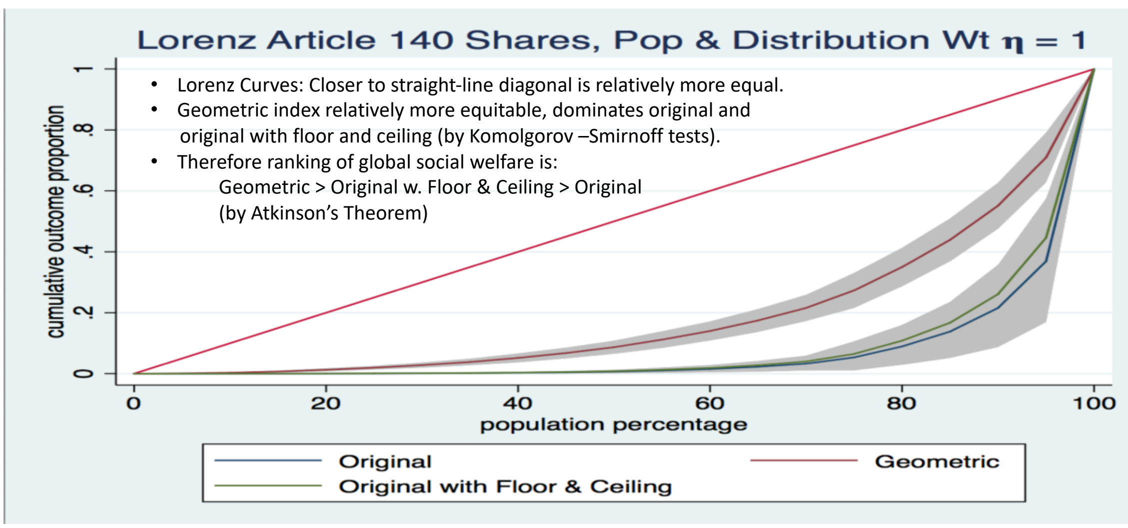


[10] Pen's Parade depicts the succession of every State Party in the ISA with each State Party's "height" (vertical height or location on the vertical axis) proportional to its allocated share S_i, and ordered from the lowest to highest. States Parties are thus lined up in order of their "height" or magnitude of S_i from "shortest" to "highest". States Parties with the smallest allocated share S_i are first in line (furthest to the left and lowest in "height" in the parade) and States Parties with the highest allocated share S_i are last in line (furthest to the right in the parade). The States Party with the average S_i is endowed with average "height" or average allocated share S_i, 0.005988. The States Parties in the parade march past in some given time interval and the sight we see is presented by the curve in Figure 3. The parade shows a parade of States Parties with small distributions, and then some giants toward the very end of the parade.

Looking at the lower end of the distribution (small number of States Parties with allocated shares), it is hard to see a great divergence of the allocated shares for the three formulae due to the low degree of resolution. After about 50 percent of the States Parties have been allocated shares, a divergence is depicted in Figure 3. That is, the geometric mean formula starts to diverge from the original and original with floor and ceiling formulae. After around 90 percent of the States Parties have received allocated shares, the original and original with floor and ceiling formulae receive larger allocated shares than the geometric mean formula. That is, as depicted by the histogram (Figure 1) and kernel density estimator (Figure 2), the geometric mean takes longer to reach smaller large shares. After almost all States Parties have been allocated shares (the far right-hand side of the figure), the original formula clearly has the highest allocated share followed by the original with floor and ceiling formula followed by the geometric mean formula.

Figure 4. Lorenz Curve for Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae[11]

ARTICLE 140 ! $= 1$, LINEAR, LINEAR WITH FLOOR & CEILING, & GEOMETRIC



The Lorenz curve in Figure 4 clearly shows that the geometric mean formula (red) has the most equitable distribution followed by the original with floor and ceiling formula (green) followed by the original formula.[12] The geometric mean formula allocation begins to diverge from the other two formula

[11] The Lorenz curve depicts income inequality by comparing it to the straight diagonal line, which represents perfect equality in allocated share S_I distribution. The Lorenz curve, which lies beneath the diagonal line, shows the actual distribution. The wider the disparity between the diagonal line and the Lorenz curve, the greater the disparity in allocated shares among States Parties. Appendix 2 provides more discussion of the Lorenz curve.
[12] A Kolmogorov-Smirnov type test statistic based on the largest positive difference shows that the geometric mean Lorenz Curve differs from the original Lorenz Curve (KS Test Statistic [p-value] = 6.52e+00 [0.0000]). The same test show that the original and original with floor and ceiling differ (KS Test Statistic [p-value] = 6.52e+00 [0.0000]). The same test shows that the geometric mean Lorenz Curve differs from the original with floor and ceiling Lorenz Curve (KS Test Statistic [p-value] = 6.32e+00 [0.0000]).

after around 10-12 percent of the States Parties receive an allocation. The original with floor and ceiling formula allocation begins to diverge from the original formula allocation only after about 55 percent of the States Parties receive an allocation. The divergence between the original and original with floor and ceiling formulae allocations narrows after almost all of the States Parties receive allocations.

Table 2 measures the relative inequality for the three alternative allocation formulae using the Atkinson and Generalized Entropy (Theil) inequality measures, the Gini coefficient, the ratio of the 90th to 10th percentiles, and skewness measure for the distribution. Appendix 5 discusses each of the relative inequality measures in detail. The Atkinson measure ranges between 0 and 1, with lower values indicating greater equality and social welfare. The Generalized Entropy (Theil) measures range between 0 and infinity, with lower values indicating greater equality and greater social welfare. The Gini coefficient ranges between 0 and 1, with lower values indicating greater equality.

Table 2's rows correspond to the three different types of allocation formulae: (1) the original, (2) geometric mean, and (3) original with floor and ceiling. Each column gives a type of relative inequality measure and the ranking of each allocation formulae by that measure of relative inequality. For example, the second row is for the original allocation formula and the second column is for the Atkinson inequality index with the inequality aversion parameter $\gamma = 0.5$. The geometric mean formula has the lowest relative inequality of the distribution (indicated by the number 1 in parenthesis under the actual Atkinson measure of 0.33077), the original with floor and ceiling formula has the second lowest relative inequality (indicated by the number 2 in parenthesis under the Atkinson value of 0.64778), and original formula has the most inequitable distribution indicated by the number 3 in parenthesis under the Atkinson value of 0.69352).

Every single measure of relative inequality with $\eta = 1$ indicates that the geometric mean formula is the most equitable, followed by the original formula with floor and ceiling, and followed by the original formula. The same values and results are obtained whether the inequality measures are applied to allocated shares S_i or an actually allocation, i.e. $F_i = S_i E$, where F_i the actual dollar amount allocated to States Party i and E is the total amount of royalties to be allocated among the 167 States Parties. This result is consistent with the Lorenz curve depicted in Figure 4.

Table 2. Measures of Relative Inequality and Social Welfare for the Allocated Shares or Actual Allocation for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae, $\eta = 1$

Type of Allocated Share S_i	Atkinson Inequality Index $A(\gamma)$ $\gamma = 0.5$	Atkinson Inequality Index $A(\gamma)$ $\gamma = 1$	Atkinson Inequality Index $A(\gamma)$ $\gamma = 2$	Gini Coefficient w. Standard Error	Theil's First (T) Inequality Index GE(1)	Theil's Second (L) Inequality Index GE(0)	Percentile Ratio p90/p10	Skewness $\eta = 1$
Linear	0.69532 (3)	0.94910 (3)	0.99948 (3)	0.8694 (SE=0.0469) (3)	1.97984 (3)	2.97793 (3)	2385.228 (3)	10.11 (3)
Geometric	0.33077 (1)	0.59128 (1)	0.87892 (1)	0.6264 (SE=0.0285) (1)	0.71947 (1)	0.89472 (1)	48.839 (1)	3.92 (1)
Linear with Floor (0.00001) and Ceiling (0.1631)	0.64778 (2)	0.92465 (2)	0.99063 (2)	0.8431 (SE=0..0249) (2)	1.62406 (2)	2.58561 (2)	1454.887 (2)	5.82 (2)

Notes: Each column includes inequality measure and inequality rank for that column.
Atkinson: Lower values more equal, 0≤ A($\boldsymbol{\gamma}$) ≤1. Higher γ is higher inequality aversion.
Generalized Entropy:

Theil T = GE(1) & Theil L = GE(0): 0 ≤ GE(0), GE(1) ≤ ∞, lower values are more equal.
GE(1) more sensitive to higher income than GE(0).

With positive and large α, the index *GE* will be more sensitive to what happens in the upper tail of the income distribution.
With positive and small α, the index G*E* will be more sensitive to what happens at the bottom tail of the income distribution.
Gini coefficient: Lower values are more equal, 0 ≤ G ≤ 1. Standard error given in parentheses.
All values equivalent for allocated share $0 \leq S_i \leq 1$ and allocated share of US$500 million.

5. Equitable Distribution for States Parties with Per Capita GNIs Less Than Mean Per Capita GNI for All ISA States Parties

The following table, Table 3, and figure, Figure 5, examine the relative inequality of the three allocation formulae for States Parties with per capita GNIs that are less than the mean per capita GNI for all ISA States Parties with $\eta = 1$. This criterion is equivalent to a social distribution weight greater than one: $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} > 1$.

The results in Table 3 and Figure 5 show that the geometric mean allocation formula provides the most equitable allocation when $\eta = 1$.

Table 3 evaluates the relative inequality of the three allocations for the States Parties with per capita GNIs that are less than the mean per capita GNI for all ISA States Parties with $\eta = 1$. The results are consistent with the relative inequality calculated over all States Parties, presented in Table 3. Hence, the geometric allocation formula is most equitable for any degree of inequality aversion.

Table 3. Measures of Relative Inequality and Social Welfare for the Allocated Shares or Actual Allocation for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae: Per Capita GNI < Global Mean per Capita GNI $(\omega_i > 1), \eta = 1$

Formula/Measure	Atkinson Inequality Index $A(\gamma)$ $\gamma = 1$	Atkinson Inequality Index $A(\gamma)$ $\gamma = 2$	Theil's Second (L) Generalized Entropy Inequality Index GE(0)	Theil's First (T) Generalized Entropy Inequality Index GE(1)	Gini Coefficient
Original	0.91637 (3)	0.99827 (3)	2.48130 (3)	1.69906 (3)	0.8295325 (3)
Geometric	0.52667 (1)	0.83916 (1)	0.74796 (1)	0.58325 (1)	0.5716104 (1)
Original with Floor and Ceiling (0.1631)	0.88677 (2)	0.99021 (2)	2.17833 (2)	1.34792 (2)	0.7951952 (2)

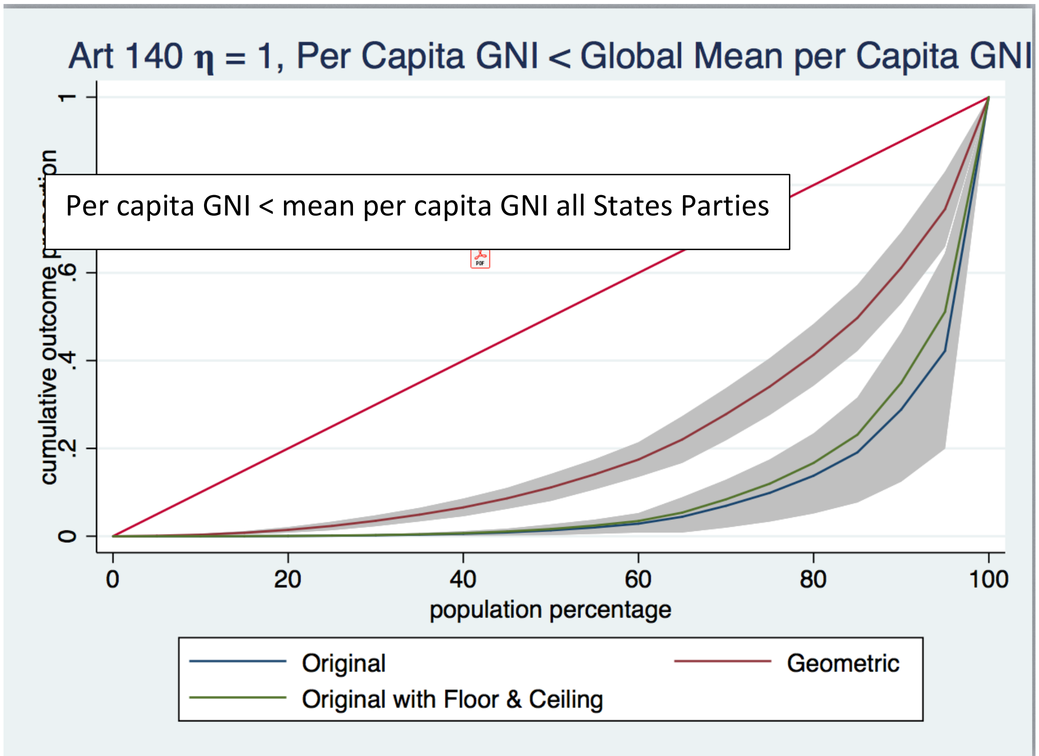
Notes: Each column includes inequality measure and inequality rank for that column.
Atkinson: Lower values more equal, 0≤ A(γ) ≤1. Higher γ is higher inequality aversion.
Generalized Entropy:

Theil T = GE(1) & Theil L = GE(0): 0 ≤ GE(0), GE(1) ≤ ∞, lower values are more equal.

GE(1) more sensitive to higher income than GE(0).

With positive and large α, the index *GE* will be more sensitive to what happens in the upper tail of the income distribution.
With positive and small α, the index G*E* will be more sensitive to what happens at the bottom tail of the income distribution.
Gini coefficient: Lower values are more equal, 0 ≤ G ≤ 1. Standard error given in parentheses.
All values equivalent for allocated share $0 \leq S_i \leq 1$ and allocated share of US$500 million.

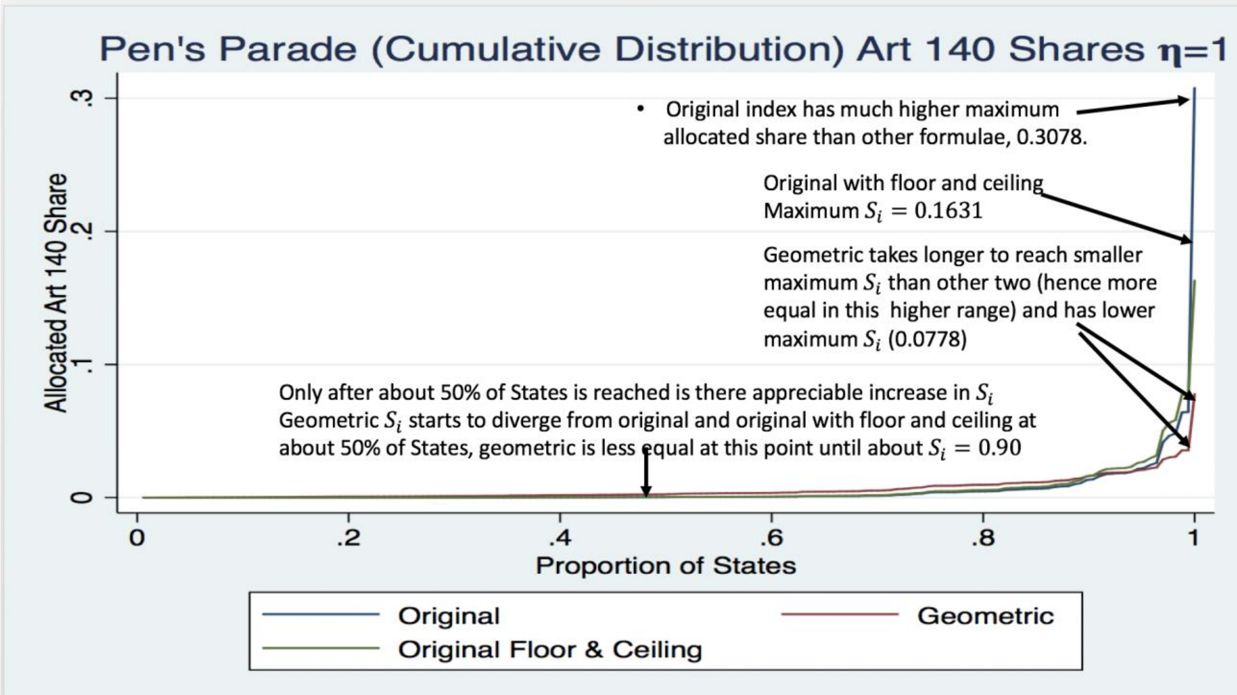
Figure 5. Lorenz Curve for Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae $\eta = 1$: Per Capita GNI < Global Mean per Capita GNI



The Lorenz Curve for the three allocation formulae for the States Parties whose per capita GNI is less than the mean global per capita GNI for all States Parties shows that the geometric mean index clearly gives a more equitable distribution and that the original and original with floor and ceiling give very close results to one another except at the lowest allocation shares (as expected due to the floor and the inapplicability of the ceiling).[13] Note that because $\eta = 1$, the analysis and results are equivalent to evaluate on the basis of the social distribution weights ω_i.

[13] A Kolmogorov-Smirnov type test statistic based on the largest positive difference shows that the geometric mean Lorenz Curve differs from the original Lorenz Curve (KS Test Statistic [p-value] = 6.52e+00 [0.0000]). The same test show that the original and original with floor and ceiling differ (KS Test Statistic [p-value] = 6.52e+00 [0.0000]). The same test shows that the geometric mean Lorenz Curve differs from the original with floor and ceiling Lorenz Curve (KS Test Statistic [p-value] = 6.32e+00 [0.0000]).

Figure 6. Pen's Parade for Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling Formulae $\eta = 1$: Per Capita GNI < Global Mean per Capita GNI



Pen's Parade in Figure 6 clearly shows that the geometric mean allocation formula allocates larger shares to more States Parties with per capita GNI less than the mean per capita GNI than do the original formula or the original with floor and ceiling.

6. Distribution of Allocated Shares by ISA Regions

The allocated shares S_i for the original formula $\eta = 1$, ranked by size from largest to smallest is, as indicated by Table 4, is:

1. Africa (Africa) 28.144%
2. Asia-Pacific Group (APG) 26.946%
3. Latin American and Caribbean Group (GRULAC) 17.356%
4. Eastern European Group (EEG) 13.722%
5. Western European and Other Group (WEOG) 13,722%.

Table 4. Summary Statistics of Allocated Shares by Region for Original Formula $\eta = 1$

REGION	Popn. share	Mean	__00000F	Income share	log(mean)
APG	**0.26946**	**0.01282**	**2.14122**	**0.57697**	**-4.35662**
Africa	**0.28144**	**0.00803**	**1.34067**	**0.37731**	**-4.82482**
EEG	**0.13772**	**0.00064**	**0.10756**	**0.01481**	**-7.34773**
GRULAC	**0.17365**	**0.00092**	**0.15406**	**0.02675**	**-6.98837**
WEOG	**0.13772**	**0.00018**	**0.03009**	**0.00414**	**-8.62167**
.					

Tables 5-10 provide Atkinson and Generalized Entropy (Theil) and Gini Coefficient values for the equity of allocated shares to each ISA regional group for the Original formula (Tables 5-6), Geometric Mean formula (Tables 7-8), and Original with Floor and Ceiling formula (Tables 9-10) with $\eta = 1$. The relative rankings for each measure are given in parenthesis and outlined by red (on a column-by-column basis, for each measure, where rows give the ISA regional group). The relative rankings of equitable distribution are consistent across the Atkinson and Generalized Entropy (Theil) and Gini Coefficient values.

The equity of distribution to ISA regions depends upon heterogeneity of each region's States Parties by population share P_i and to a lesser extent the magnitude of each States Party i's social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$. The ranking of equitable distribution by ISA region from the most to least equitable distribution (where relative equity is determined by the Atkinson and Generalized Entropy measures) is:

1. Eastern European Group
2. Western European and Other Groups
3. Africa
4. Latin American and Caribbean Group
5. Asia Pacific Group

The same ranking of the distribution for social welfare is found as with the ranking of relative inequality, i.e. EEG group receives highest social welfare relative to others, WEOG next most, etc. with $\eta = 1$.

Table 5. Atkinson Inequality Values of Allocated Shares by Region: Original Formula, $\eta = 1$

Less inequality aversion. More inequality aversion.

REGION	A(0.5)	A(1)		A(2)
APG	**0.75846**	**0.98411**	(5)	**0.99990**
Africa	**0.38204**	**0.74293**	(3)	**0.98895**
EEG	**0.40294**	**0.63592**	(1)	**0.83009**
GRULAC	**0.50896**	**0.86709**	(4)	**0.98700**
WEOG	**0.34305**	**0.69423**	(2)	**0.99087**
.				
Pacific Island Developing	0.69532	0.94910.		0.99948

Atkinson: Lower values more equal, $0 \leq A(\gamma) \leq 1$.

Table 6. Generalized Entropy (Theil) Inequality Values and Gini Coefficient of Allocated Shares by Region: Original Formula, $\eta = 1$

	More sensitive to low income				More sensitive to high income
REGION	GE(-1)	GE(0)	GE(1)	GE(2)	Gini
APG	5.20e+03	4.14187 (5)	2.06716 (5)	6.59926	0.89425 (5)
Africa	44.73293	1.35840 (3)	0.76718 (2)	1.09034	0.64294 (2)
EEG	2.44274	1.01038 (1)	0.98184 (3)	1.76314	0.68547 (3)
GRULAC	37.95828	2.01812 (4)	1.04958 (4)	1.50736	0.73359 (4)
WEOG	54.26158	1.18492 (2)	0.65766 (1)	0.72040	0.60414 (1)
.					
Pacific Island Developing	961.13738	2.97793	1.97984	9.08714	0.86938

Gini coefficient: Lower values more equal, 0 ≤ G ≤ 1.
Theil T = GE(1) & Theil L = GE(0): 0 ≤ GE(0), GE(1) ≤ ∞, lower values more equal.
With positive and large α, the index *GE* will be more sensitive to what happens in the upper tail of the income distribution.
With positive and small α, the index *GE* will be more sensitive to what happens at the bottom tail of the income distribution.

The next two tables, Tables 7-8 present the Atkinson, Generalized Entropy (Theil), and Gini Coefficient inequality measures for the geometric mean formula with $\eta = 1$. The results are consistent with the original formula with $\eta = 1$.

Table 7. Atkinson Inequality Values of Allocated Shares by Region: Geometric Mean Formula, $\eta = 1$

	Less inequality aversion.		More inequality aversion.
REGION	A(0.5)	A(1)	A(2)
APG	0.44150	0.74349 (5)	0.94780
Africa	0.16616	0.35502 (3)	0.70472
EEG	0.11976	0.21911 (1)	0.36744
GRULAC	0.24701	0.47975 (4)	0.75054
WEOG	0.14500	0.31777 (2)	0.70734
.			
Pacific Island Developing	0.33077	0.59128.	0.87892

Atkinson: Lower values more equal, 0≤ A(γ) ≤1.

Table 8. Generalized Entropy (Theil) Inequality Values and Gini Coefficient of Allocated Shares by Region: Geometric Mean Formula, $\eta = 1$

	More sensitive to low income				More sensitive to high income
REGION	GE(-1)	GE(0)	GE(1)	GE(2)	Gini
APG	9.07877	1.36057 (5)	0.97262 (5)	1.57007	0.71252 (5)
Africa	1.19331	0.43853 (3)	0.30831 (3)	0.30912	0.42591 (3)
EEG	0.29044	0.24732 (1)	0.26041 (1)	0.33743	0.38097 (1)
GRULAC	1.50434	0.65345 (4)	0.47993 (4)	0.51824	0.53108 (4)
WEOG	1.20844	0.38239 (2)	0.26810 (2)	0.26109	0.39939 (2)
.					
Pacific Island Developing	3.62952	0.89472	0.71947	1.14107	0.62635

Gini coefficient: Lower values more equal, 0 ≤ G ≤ 1.
Theil T = GE(1) & Theil L = GE(0): 0 ≤ GE(0), GE(1) ≤ ∞, lower values more equal.
With positive and large α, the index *GE* will be more sensitive to what happens in the upper tail of the income distribution.
With positive and small α, the index *GE* will be more sensitive to what happens at the bottom tail of the income distribution.

The next two tables, Tables 9-10 present the Atkinson, Generalized Entropy (Theil), and Gini Coefficient inequality measures for the original with floor and ceiling formula with $\eta = 1$. The results are consistent with the original and geometric mean formulae with $\eta = 1$.

Table 9. Atkinson Inequality Values of Allocated Shares by Region: Original with Floor and Ceiling Formula, $\eta = 1$

	Less inequality aversion.		More inequality aversion.
REGION	A(0.5)	A(1)	A(2)
APG	0.71150	0.96787 (5)	0.99673
Africa	0.38150	0.73567 (3)	0.97622
EEG	0.40294	0.63592 (1)	0.83009
GRULAC	0.49765	0.83461 (4)	0.96901
WEOG	0.31409	0.57418 (2)	0.81667
.			
Pacific Island Developing	0.65558	0.92691	0.99089

Atkinson: Lower values more equal, 0≤ A(γ) ≤1.

Table 10. Generalized Entropy (Theil) Inequality Values and Gini Coefficient of Allocated Shares by Region: Original with Floor and Ceiling Formula, $\eta = 1$

	More sensitive to low income				More sensitive to high income
REGION	GE(−1)	GE(0)	GE(1)	GE(2)	Gini
APG	142.15783	3.37551 (5)	1.67107 (5)	3.42355	0.85827 (5)
Africa	20.61012	1.33073 (3)	0.76704 (2)	1.09028	0.64291 (2)
EEG	2.44274	1.01038 (2)	0.98184 (3)	1.76314	0.68547 (3)
GRULAC	15.68981	1.80056 (4)	1.04133 (4)	1.50189	0.73150 (4)
WEOG	2.24184	0.85519 (1)	0.63137 (1)	0.70553	0.59507 (1)
.					
Pacific Island Developing	52.87832	2.58561	1.62406	4.16830	0.84305

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Tables

Figures

APPENDIX 1. Individual Histograms and Kernal Density Estimator for Each Allocation Formula

Figure A1.1. Histogram and. Kernal Density Estimator: Original Formula $\eta = 1$

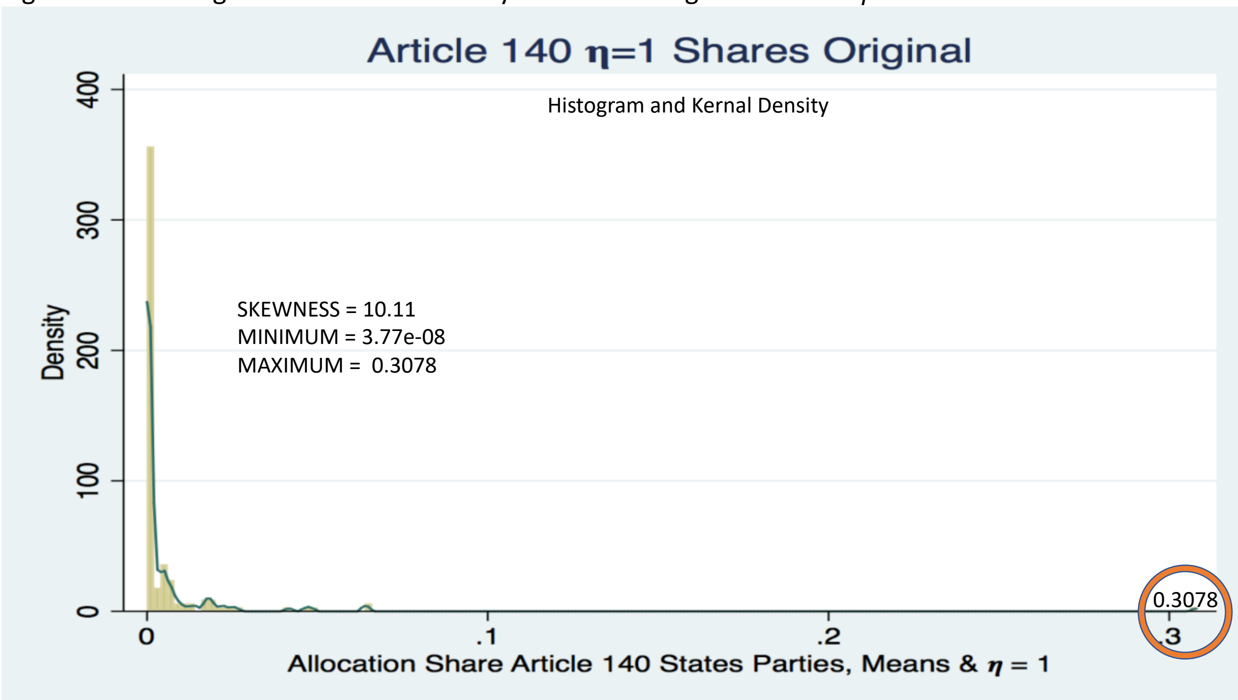


Figure A1.2. Histogram and. Kernal Density Estimator: Geometric Mean Formula $\eta = 1$

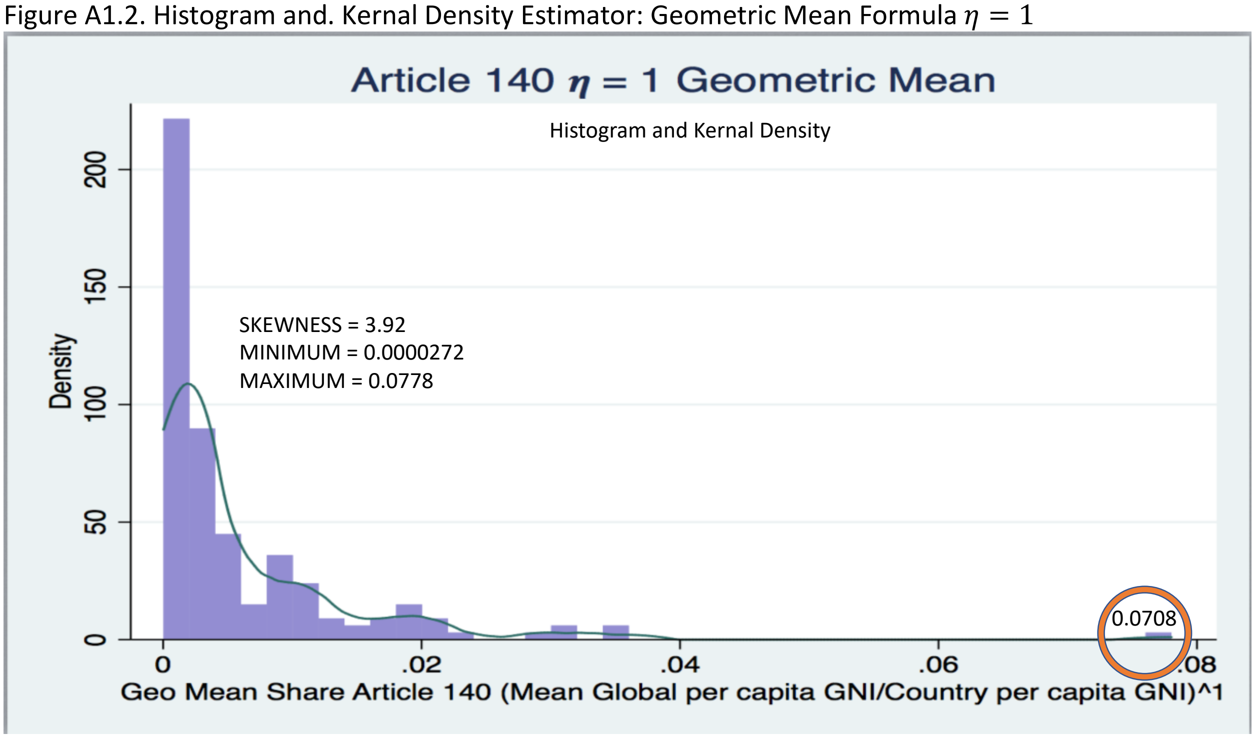


Figure A1.3. Histogram and. Kernal Density Estimator: Original with Floor and Ceiling $\eta = 1$

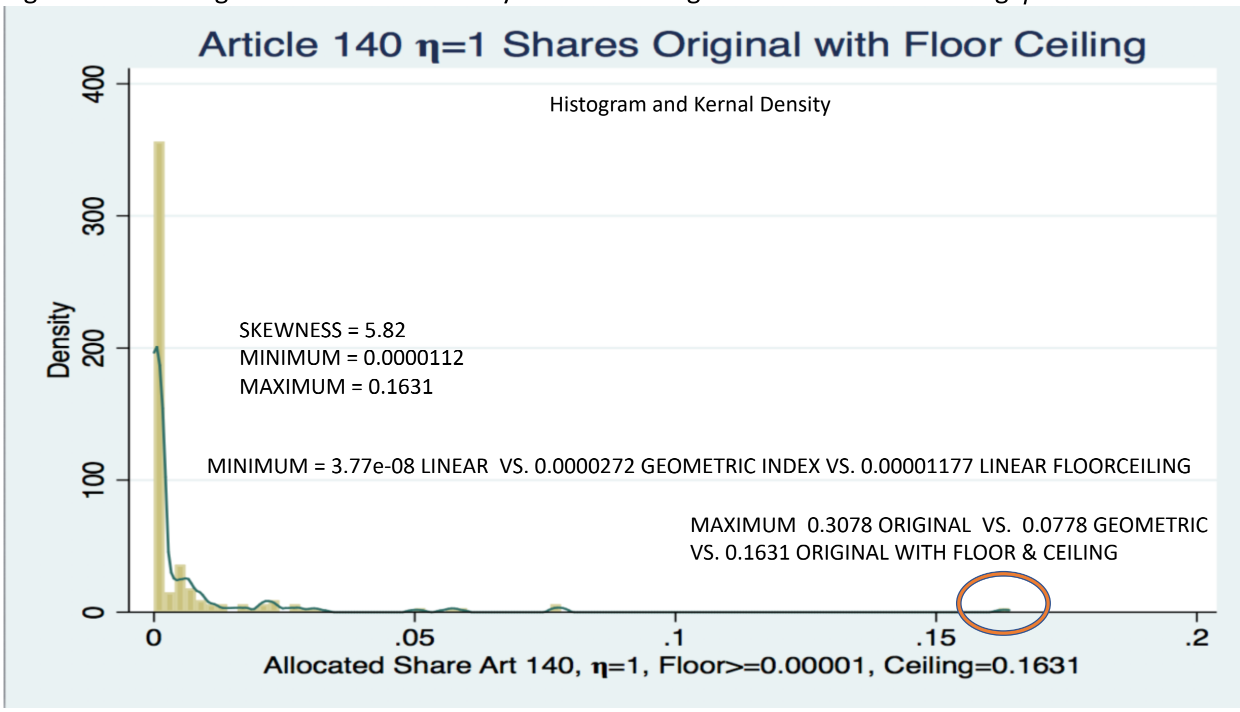


Table A1.1. Summary Statistics of the Original Formula $\eta = 1$

Percentile (%)	Arithmetic Mean	Smallest
1%	7.82e-08	3.77e-08
5%	2.58e-06	7.82e-08
10%	5.78e-06	3.21e-07
25%	0.000396	3.57e-07
50%	0.0003333	
		Largest
75%	0.0040058	0.04823
90%	0.013775	0.0640389
95%	0.022318	0.0643023
99%	0.0643023	0.3078352

Overall statistics: Arithmetic mean 0.005988, standard deviation 0.0256045, skewness 10.11276, kurtosis 117.4916.

Table A1.2. Summary Statistics by Percentile of the Geometric Mean Formula $\eta = 1$

Percentile (%)	Arithmetic Mean	Smallest
1%	0.0000392	0.0000272
5%	0.000225	0.0000392
10%	0.0003369	0.0000794
25%	0.0008821	0.0000838
50%	0.0025596	
		Largest
75%	0.0088729	0.0307878
90%	0.0164538	0.0354766
95%	0.0209434	0.03555495
99%	0.0355495	0.077782

Overall statistics: Arithmetic mean 0.005988, standard deviation 0.0090732, skewness 3.92425, kurtosis 26.44782.

Table A1.3. Summary Statistics by Percentile of the Original with Floor and Ceiling Formula $\eta = 1$

Percentile (%)	Arithmetic Mean	Smallest
1%	0.0000114	0.0000112
5%	0.0000114	0.0000114
10%	0.0000114	0.0000114
25%	0.0000479	0.0000114
50%	0.000403	
		Largest
75%	0.0048425	0.0583045
90%	0.0166524	0.0774157
95%	0.0269798	0.077734
99%	0.077734	0.1631

Overall statistics: Arithmetic mean 0.005988, standard deviation 0.0173, skewness 5.75, kurtosis 45.65.

APPENDIX 2. Impact of $\eta = 2$ Upon Original and Geometric Mean Formulae

Table A2.1 Summary Statistics of Allocated Shares by Percentile of the Original Formula $\eta = 2$

Percentile (%)	Arithmetic Mean	Smallest
1%	2.05E-09	3.72E-10
5%	7.59E-08	2.05E-09
10%	3.26E-07	1.70E-08
25%	2.01E-06	1.85E-08
50%		
		Largest
75%	0.0007662	0.0403341
90%	0.0085169	0.1228704
95%	0.0262127	0.1674672
99%	0.1674672	0.283344

Overall statistics: Arithmetic mean 0.005988, standard deviation 0.0276168, skewness 7.820978, kurtosis 70.72772.

Table A2.2. Summary Statistics of Allocated Shares by Percentile of the Geometric Mean Formula $\eta = 2$

Percentile (%)	Arithmetic Mean	Smallest
1%	8.07e-06	3.44e-06
5%	0.000491	
10%	0.0001017	
25%	0.0002529	
50%	0.0011646	
		Largest
75%	0.0049322	0.0357857
90%	0.0164442	0.0624592
95%	0.0288489	0.0729185
99%	0.0729185	0.0948483

Overall statistics: Arithmetic mean 0.005988, standard deviation 0.0124577, skewness 4.112711, kurtosis 24.25342.

Table A2.3. Summary Statistics by Percentile of Percentage Difference in Allocated Shares for the Original Formulae $\eta = 2$ Minus Allocated Shares for the Original Formulae $\eta = 1$

Percentile (%)	Arithmetic Mean	Smallest
1%	-99.16	-99.53
5%	-98.46	-99.16
10%	-98.21	-99.05
25%	-94.62	-98.99
50%	-85.56	
		Largest
75%	-55.42	103.13
90%	1.34	107.79
95%	42.36	128.39
99%	128.39	500.42

Overall statistics: Arithmetic mean -63.19%, standard deviation 67.89, skewness 5.75, kurtosis 50.40.

Table A2.4. Summary Statistics by Percentile of Percentage Difference in Allocated Shares for the Geometric Mean Formulae $\eta = 2$ Minus Allocated Shares for the Geometric Mean Formulae $\eta = 1$

Percentile (%)	Arithmetic Mean (%)	Smallest (%)
1%	-88.25	-91.24
5%	-84.24	-88.25
10%	-83.01	-87.59
25%	-70.53	-87.26
50%	-51.70	
		Largest (%)
75%	-15.13	81.15
90%	27.95	83.22
95%	51.65	92.06
99%	92.09	231.55

Overall statistics: Arithmetic mean -38.57, standard deviation 46.76, skewness 1.95, kurtosis 9.10.

Table A2.5. Atkinson and Generalized Entropy (Theil) Inequality Measures and Gini Coefficient Original and Geometric Mean Formulae $\eta = 1$ and $\eta = 2$

Formula/Measure	Atkinson's Inequality Index $A(\gamma = 0)$	Atkinson's Inequality Index $A(\gamma = 1)$	Atkinson's Inequality Index $A(\gamma = 2)$	Theil's Second (L) Generalized Entropy Inequality Index $GE(0)$	Theil's First (T) Generalized Entropy Inequality Index $GE(1)$	Gini Coefficient	P90/P10
Original $\eta = 1$	0.69532 (3)	0.94910 (3)	0.99948 (4)	2.97793 (3)	1.97984 (3)	0.86938 (3)	2385.228 (3)
Original $\eta = 2$	0.81140 (4)	0.9999 (4)	0.99279 (3)	4.93166 (4)	2.41495 (4)	0.92215 (4)	26149.12 (4)
Geometric Mean $\eta = 1$	0.33077 (1)	0.59128 (1)	0.87892 (1)	0.89472 (1)	0.71947 (1)	0.62635 (1)	48.839 (1)
Geometric Mean $\eta = 2$	0.50366 (2)	0.80442 (2)	0.97190 (2)	1.63176 (2)	1.15597 (2)	0.75735 (2)	161.707 (2)

Notes: Each column includes inequality measure and inequality rank for that column.
Atkinson: Lower values more equal, $0 \leq A(\boldsymbol{\gamma}) \leq 1$. Higher γ is higher inequality aversion.
Generalized Entropy:

Theil T = GE(1) & Theil L = GE(0): 0 ≤ GE(0), GE(1) ≤ ∞, lower values are more equal.

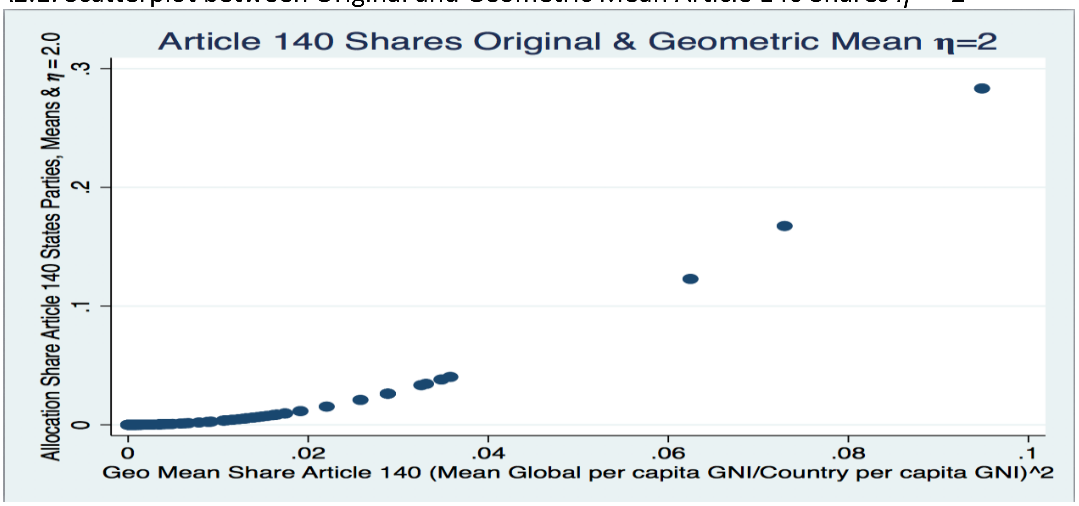
GE(1) more sensitive to higher income than GE(0).

With positive and large α, the index G*E* will be more sensitive to what happens in the upper tail of the income distribution.
With positive and small α, the index G*E* will be more sensitive to what happens at the bottom tail of the income distribution.
Gini coefficient: Lower values are more equal, 0 ≤ G ≤ 1. Standard error given in parentheses.
All values equivalent for allocated share $0 \leq S_i \leq 1$ and allocated share of US$500 million.

The Atkinson and Generalized Entropy (Theil) inequality measures, Gini Coefficient, and ratio of the top 90th percentile to the lower 10th percentile allocated shares S_i when $\eta = 2$ (almost always) shows the following rankings from the distribution that is most equitable with highest social welfare: geometric mean $\eta = 1$ > geometric mean $\eta = 2$ > original $\eta = 1$ > original $\eta = 2$.

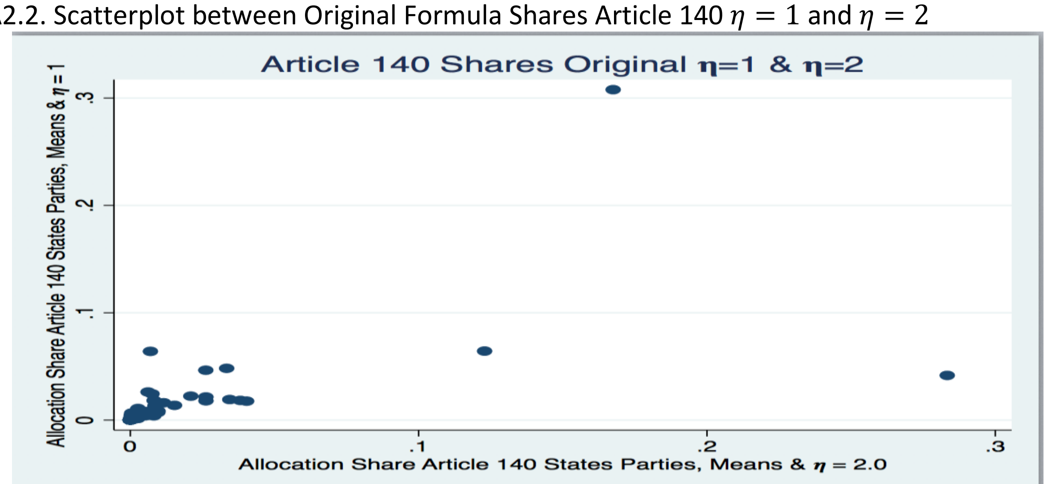
The scatterplot in Figure A2.1. between the original (vertical axis) and geometric mean (horizontal axis) formulae Article 140 shares S_i with $\eta = 2$ is somewhat nonlinear that increases at an increasing rate. The scatterplot also shows that the largest three States Parties' shares dominate the relationship that increases at an increasing rate.

Figure A2.1. Scatterplot between Original and Geometric Mean Article 140 Shares $\eta = 2$



The scatterplot in Figure A2.2. between the original Article 140 shares S_i with $\eta = 1$ (vertical axis) and original Article 140 shares S_i with $\eta = 2$ (horizontal axis) shows that shares are positively related and concentrated with low share values. However, the large shares of three States Parties again are much larger than for other States Parties with one share showing an increase at a decreasing rate and one share showing an increase at a decreasing rate.

Figure A2.2. Scatterplot between Original Formula Shares Article 140 $\eta = 1$ and $\eta = 2$



The scatterplot in Figure A2.3. between the original Article 140 shares S_i with $\eta = 1$ (vertical axis) and original Article 140 shares S_i with $\eta = 2$ (horizontal axis) shows that shares are positively related and concentrated with low share values. However, the large shares of three States Parties again are much larger than for other States Parties with one share showing an increase at a decreasing rate and one share showing an increase at a decreasing rate.

The same pattern for the three largest shares holds for both the original and geometric mean formulae. However, the geometric mean shares are less concentrated and has a wider dispersion.

Figure A2.3. Scatterplot between Geometric Mean Formula Shares Article 140 $\eta = 1$ and $\eta = 2$

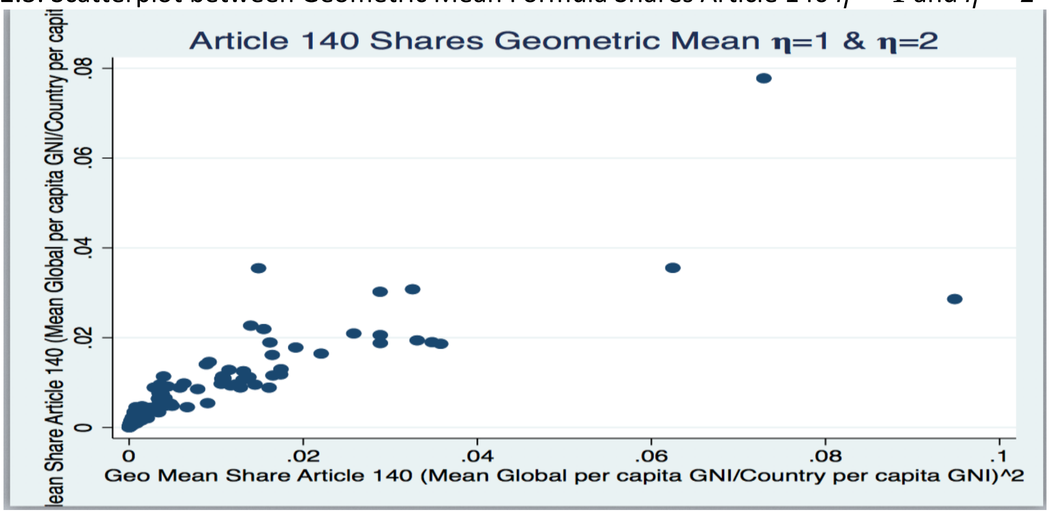


Figure A2.4. Histogram for Percentage Difference in Social Distribution Weights for the Original Formula $\eta = 2 - \eta = 1$

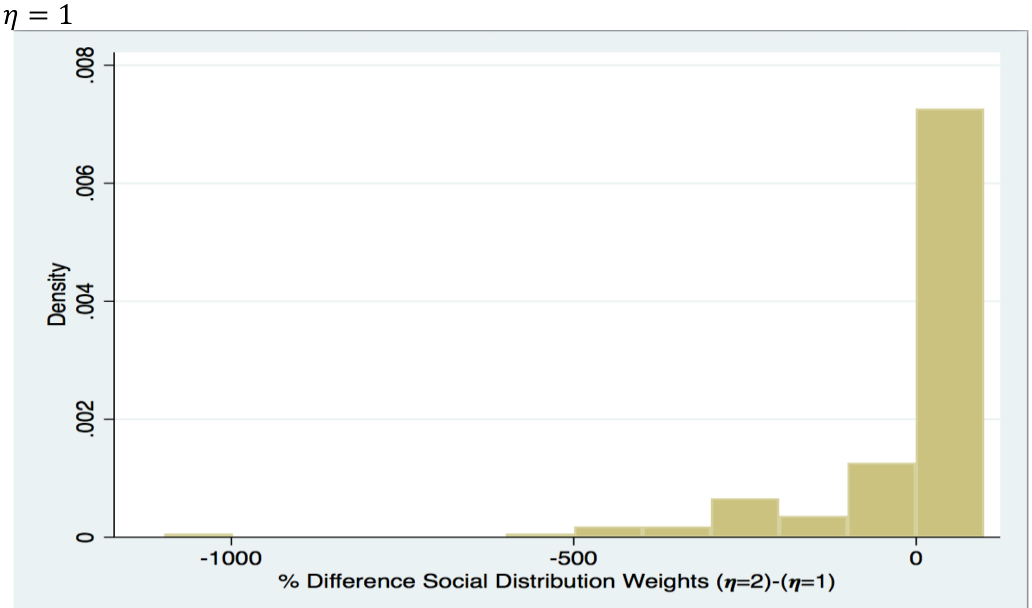


Figure A2.5. Histogram and Kernel Density Estimator of (Difference in) Social Distribution Weights for the Original Formulae $\eta = 2$ Minus Social Distribution Weights for the Original Formulae $\eta = 1$

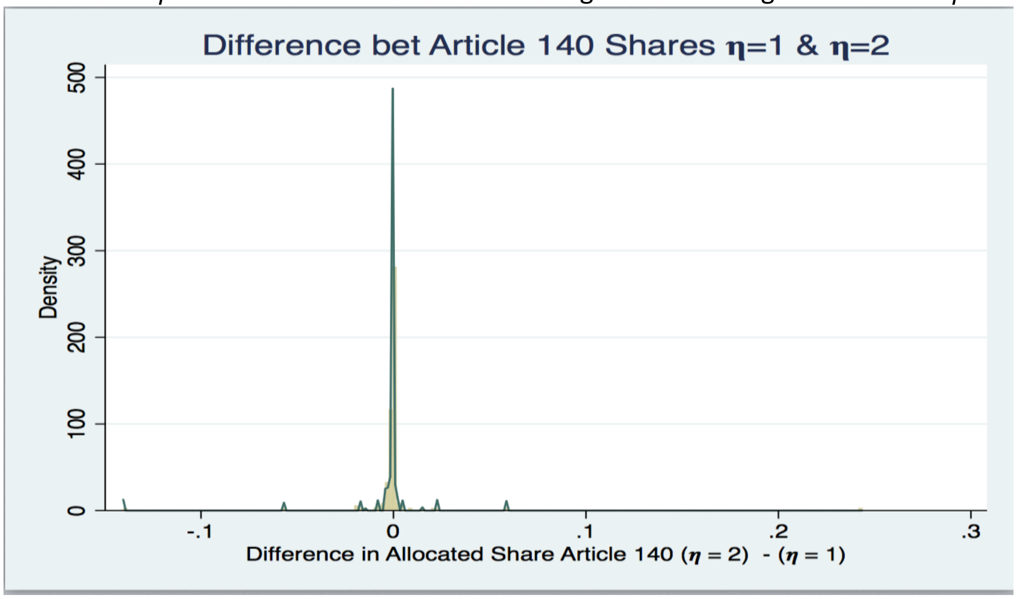


Figure A2.6. Histogram of Allocated Shares for the Original Formulae $\eta = 1$ and $\eta = 2$

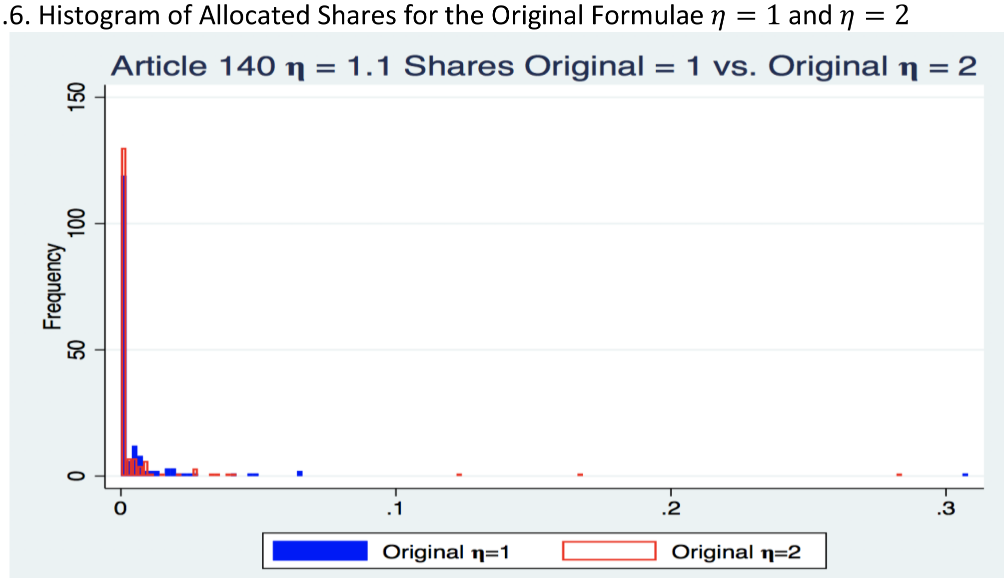


Figure A2.7. Kernel Density Estimator of Allocated Shares for the Original Formulae $\eta = 1$ and $\eta = 2$

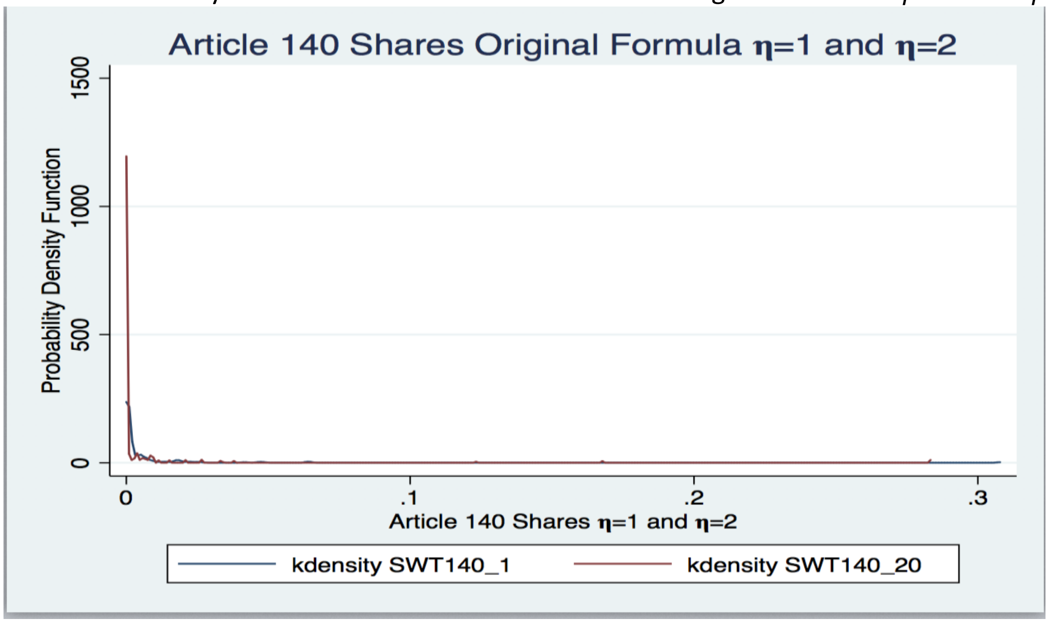
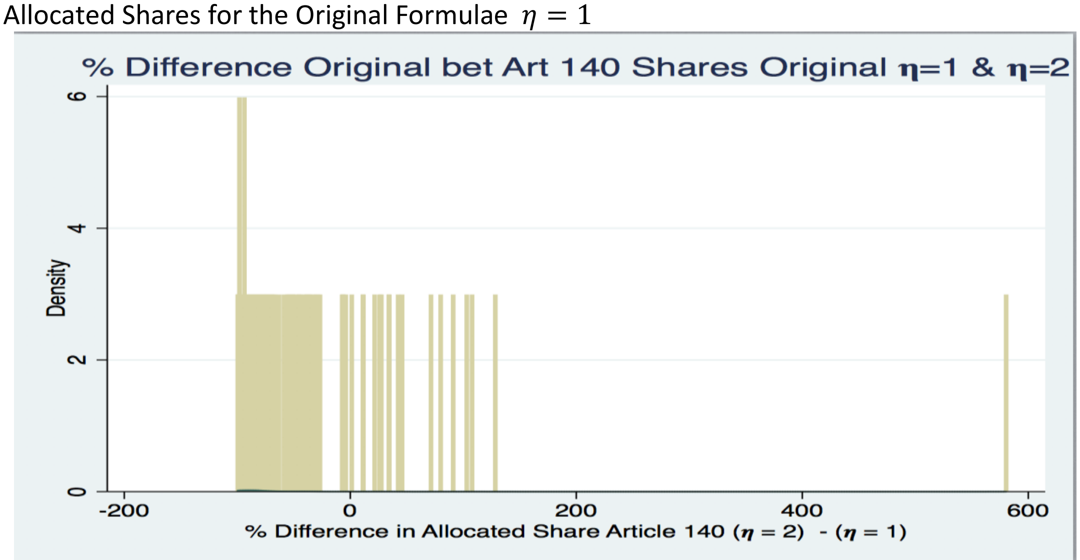


Figure A2.8. Histogram of Percentage Difference in Allocated Shares for the Original Formulae $\eta = 2$ Minus Allocated Shares for the Original Formulae $\eta = 1$



The figure clearly shows that more States Parties lose on a percentage basis with the original formula when increasing η from $\eta = 1$ to $\eta = 2$ and that a limited number of States Parties enjoy a percentage gain of almost 600 percent. The distribution is skewed with a long tail in favor of gainers and a concentration of losers.

Figure A2.9. Histogram of Allocated Shares for the Geometric Mean Formulae $\eta = 1$ and $\eta = 2$

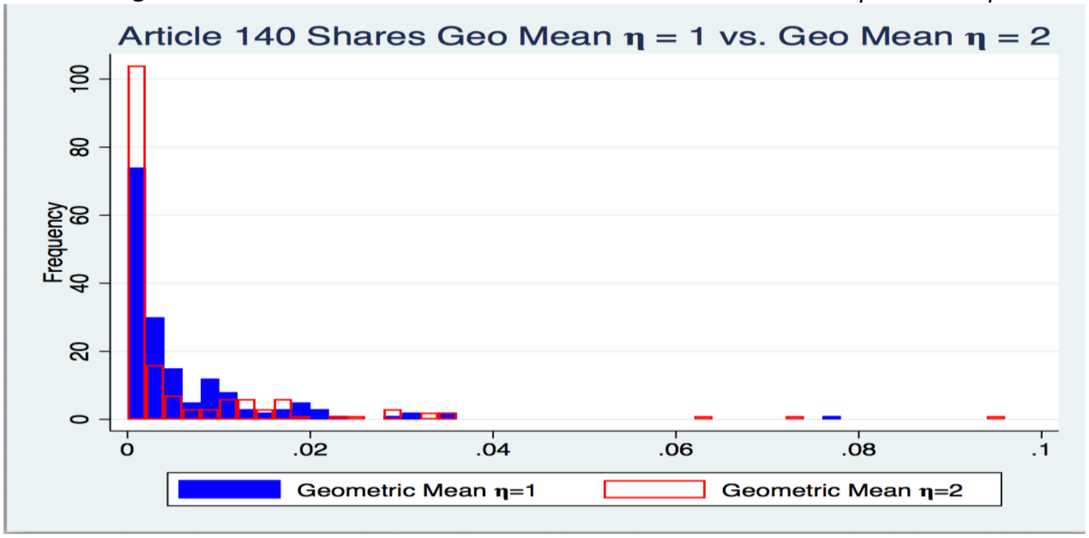
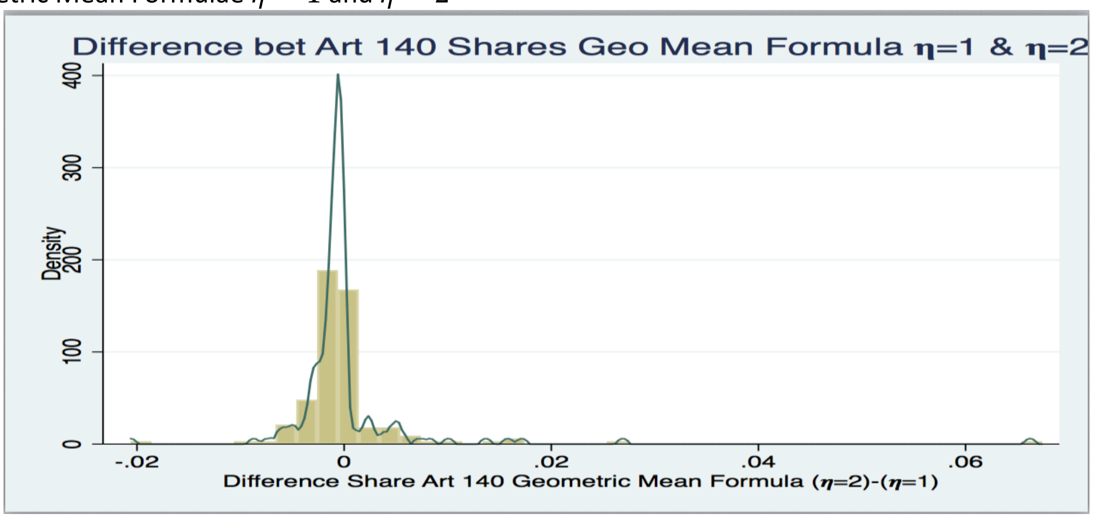
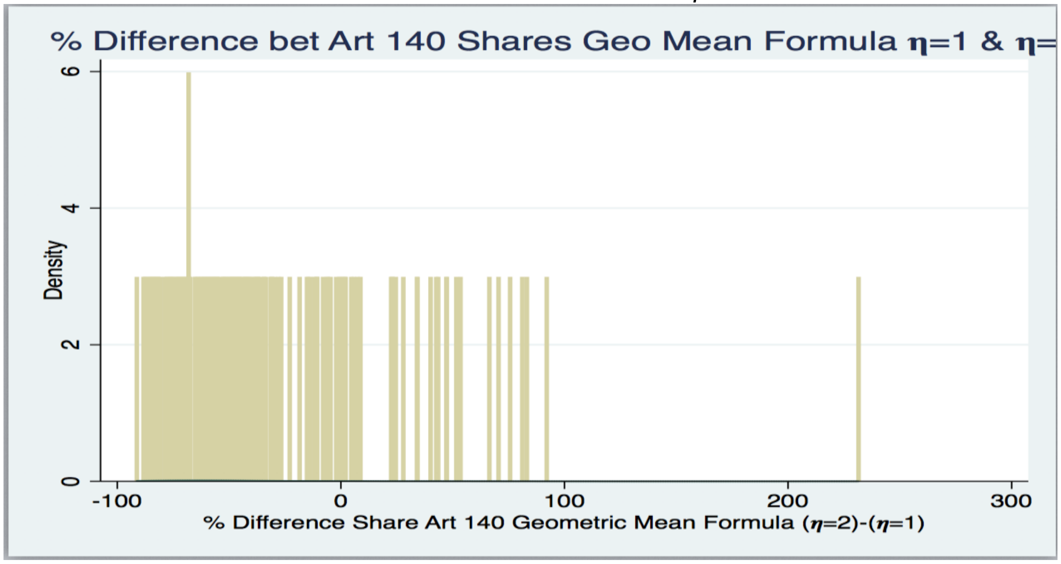


Figure A2.10. Histogram and Kernel Density Estimator of Difference in Allocated Shares for the Geometric Mean Formulae $\eta = 1$ and $\eta = 2$



The histogram and kernel density estimator for the difference between **η**=2 and **η**=1 with the geometric mean formula shows that in terms of numbers (frequency) of States Parties more States Parties lose than gain.

Figure A2.11. Histogram of Percentage Difference in Allocated Shares for the Geometric Mean Formulae $\eta = 2$ Minus Allocated Shares for the Geometric Mean Formulae $\eta = 1$



The figure clearly shows that more States Parties lose on a percentage basis with the geometric mean formula when increasing η from $\eta = 1$ to $\eta = 2$ and that a limited number of States Parties enjoy a percentage gain of over 200 percent. The distribution is skewed with a long tail in favor of gainers and a concentration of losers. Compared to the original formula, there is a shorter tail of gainers (right-hand side of the figure). and a longer tail of losers (left-hand side of the figure).

Figure A2.12. Pen's Parade for Allocated Shares for the Original and Geometric Mean Formulae $\eta = 1$ and $\eta = 2$

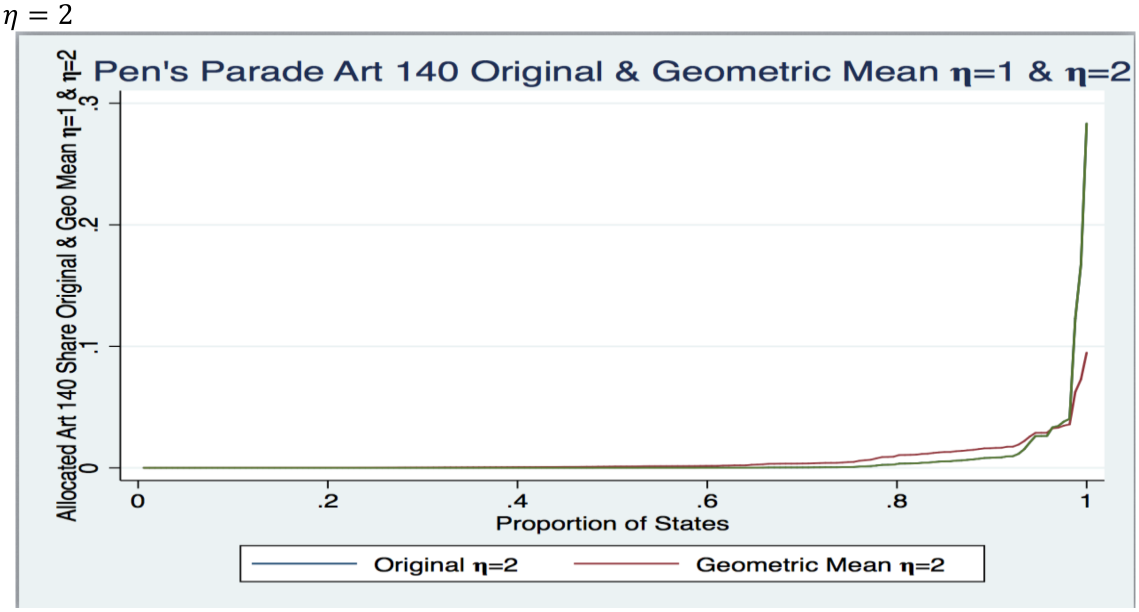
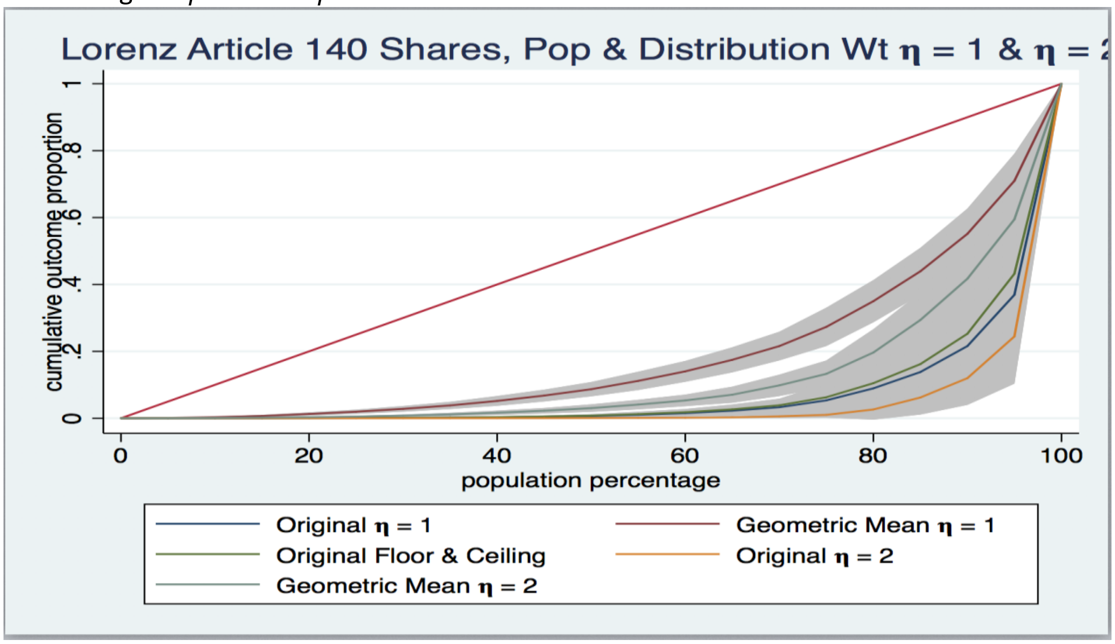


Figure A2.13. Lorenz Curve for Allocated Shares for the Original, Geometric Mean, and Original with Floor and Ceiling for $\eta = 1$ and $\eta = 2$



The Lorenz Curve results reinforce the conclusions of the histograms and kernel density estimators that raising η from $\eta = 1$ to $\eta = 2$ creates more losers than gainers when reallocating proportions or shares of a fixed amount on the basis of η.

The ranking of the original and geometric mean formulae for values of **η**=1 and **η**=2 in terms of most equitable and highest social welfare from highest to lowest is: geometric mean **η**=1 > geometric mean **η**=2 > original with floor and ceiling **η**=1 > original **η**=1 > original **η**=2.

APPENDIX 3. Alternative Approaches and Formulae Not Adopted

The Finance Committee suggested several options for the allocation formula (essentially for the numerator). Each of these has been reviewed, but it is considered that for the reasons set out below these approaches would make no meaningful difference to the allocation formula.

A.3.1. Equal Weights for Population Share P_i for each States Party. In this case. $P_i = P_j,\ i \neq j,\ \forall\, i, j\ \in N$. This formula reduces to the same formula as the State as the basic unit to represent the Common Heritage of Mankind rather than heterogenous States Parties' population shares, because multiplying each States Party's social distribution weight ω_i by the same constant number (scalar) cancels out in both the numerator and denominator of the formula for S_i. Thus:

$$S_i = \frac{P\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}}{\sum_{i=1}^{N} P\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}} = \frac{\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}}$$

A.3.2. Population Density for each States Party. An additional variable could be added to the allocated shares formula, the population density of each States Party's population, denoted D_i. The variables $P_i\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$ are thus multiplied by D_i to give for the original formula:

$$S_i = \frac{\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta} * P * D_i}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta} * P_i * D_i}$$

The impact upon the size and distribution of S_i is expected to be closely aligned to the results without D_i due to the strength of the population share of all ISA States Parties, P_i.

A.3.3. Additional Criteria would be summed to form an aggregate index of the individual criteria C_{ij} for States Party i,:

$$C_i = \prod_{j=1}^{M} C_{ij}{}^{\beta_j}$$

$i = 1,2, \ldots., N$, where N denotes the number of States Parties, $j = 1,2, \ldots., M$ individual criteria, M denotes the number of criteria, β_j denotes the weight given to individual criteria C_{ij}, and $\sum_{j=1}^{M}\beta_j = 1$. In addition to methods to obtain the stated preference weights for each individual criterion, different index number formula can be considered, where the above is the geometric mean if $\beta_j = \beta_k = \frac{1}{M}, j \neq k, \forall j, j, k\varepsilon M, 0 < \beta_j < 1, \sum_{j=1}^{M}\beta_j = 1$, i.e. β_j becomes the j^{th} root.

The formula for States Parties' allocated shares S_i can be written:[14]

[14] The index C_i can be directly constructed in the allocation formula in a single step. The index C_i can also be constructed inn two or more stages. Multi-stage versus single-stage construction raises the issue of consistency in aggregation, which is discussed in Appendix 8.

$$S_i = \frac{\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta} * C_i}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta} * C_i}.$$

A.3.4. State Party the Unit for the Common Heritage of Mankind rather Than the Individual Person. The individual State Party can be the unit for the Common Heritage of Mankind Principle and for the allocated share S_i rather than the individual person. Aristotle's Equity Principle still applies, where each State Party has an equal claim. In this case, each State Party's population share P_i is replaced by the integer 1. The original allocation formula becomes:

$$S_i = \frac{\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}}$$

There is no geometric mean formula for the State Party as the basis for the Common Heritage of Mankind, since there is only a single variable, $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}$.

$$S_i = \frac{\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}}{\sum_{i=1}^{N}\left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}}$$

APPENDIX 4. Regression Analysis of the Impact of $\boldsymbol{P_i}$ and $\boldsymbol{\omega_i}$ Upon $\boldsymbol{S_i}$

The impact of population share P_i and social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}$, $\eta = 1$, upon S_i can be evaluated by regression analysis.[15] [16] The constant term is the allocated shares to the Asia-Pacific Group, so that the dummy (categorical) variables for each Regional Group indicate deviations of that Regional Group's allocated shares from the Asia-Pacific Group's allocated shares. Standard errors are heteroscedastic-consistent and clustered around each Group (5 Groups).

The following tables show the regression results. The results show that the African Group's shares are indistinguishable from the Asia-Pacific Group's shares, but that the shares of the Latin American and Caribbean, Eastern European, and Western Europe and Others Groups are all lower. The distribution weight ω_i and share of the total population of all States Parties P_i are both statistically significant but share of the total population of all States Parties P_i has a substantially bigger impact than the distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1}$.

The average marginal effects (the effect on allocated share from a one-unit change in the independent variable, i.e. Group, population share, or distribution weight) are all statistically significant except for the African Group. The average marginal effect of the total population of all States Parties share on the size of the allocated share for each State Party P_i is 0.1084, 0.0983, 0.0765, 0.0124522 for the original, original with floor and ceiling, geometric mean, and State as the basis of the CHM formulae, respectively, and for the social distribution weight ω_i is 0.0001, 0.0001, 5.75e-07, and 0.0001498 for the original, original with floor and ceiling, geometric mean, and State as the basis of the CHM formulae, respectively (all values are always statistically significant with $P > |z| = 0.000$), indicating orders of magnitude in differential impact between P_i and ω_i, which is consistent across all three formulae.

In summary, the population shares P_i of the States Parties is the biggest single determinant of the size of the allocated Article 140 shares S_i for any formula, with the social distribution weight ω_i making a much smaller contribution by several orders of magnitude.

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[15] Proportion data have values that range between zero and one, and the predicted values should also range between zero and one. One way to accomplish this is to use a generalized linear model (glm) with a logit link and the binomial family. Standard errors in the glm model are clustered around each region to give cluster-robust standard errors, which will be particularly useful if we have mis-specified the distribution family. The Stata command is: glm SWT140_1 DAFRICA DGRULAC DEEG DWEOG POPSHARE DWT140_1, link(logit) family(binomial) cluster (REGION1) nolog. A short discussion is available at (accessed March 27, 2019): https://stats.idre.ucla.edu/stata/faq/how-does-one-do-regression-when-the-dependent-variable-is-a-proportion/

[16] The parameter estimates and standard errors are very slightly biased and inconsistent, since a fractional logit model (required when the dependent variable is proportions) does not allow the allocated shares S_i to sum to one. Nonetheless, the results clearly show the relative importance of the different variables that impact the allocated shares S_i. A similar regression with a beta distribution gives virtually identical results.

Table A4.1. Original Formula Regression Results for Impacts of Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|	95% Confidence Interval Lower	95% Confidence Interval Upper
Dummy Africa	0.219	0.247	0.09	0.376	-0.265	0.703
Dummy Latin America Caribbean	-1.429	0.010	-14.34	0.000	-1.642	-1.234
Dummy Eastern Europe	-1.739	0.096	-18.021	0.000	-1.921	-1.550
Dummy Western Europe	-2.959	0.089	-33.39	0.000	-3.135	-2.785
Population Share	19.788	0.715	27.26	0.000	18.386	21.190
Distribution Weight	0.025	0.005	5.33	0.000	0.016	0.034
Constant (Africa)	-5.700	0.090	-63.35	0.000	-5.875	-5.521

Note: Fractional logit regression by generalized linear model (glm) with a logit link and the binomial family. Robust standard errors clustered on each region (5 clusters). Intercept is Asia-Pacific Group. Log pseudolikelihood = -3.756751931, AIC = 0.0809102, BIC = -838.2343. Number of observations = 167, residual degrees of freedom = 164. Deviance =0.006089, Pearson = 0.0084279, scale parameter = 1.

Table A4.2. Marginal Impacts for Original Formula Regression Results for Impacts of ISA Regional Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|
Dummy Africa	.0011986	.001344	0.89	0.373
Dummy Latin America Caribbean	-.0078266	.0006222	-12.58	0.00
Dummy Eastern Europe	-.0095253	.0006228	-15.29	0.00
Dummy Western Europe	-.0162077	.0006498	-24.94	0.00
Population Share	.1083904	.0030193	35.90	0.00
Distribution Weight	.0001357	.0000263	5.16	0.00

Table A4.3. Geometric Mean Regression Results for Impacts of ISA Regional Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|	95% Confidence Interval Lower	95% Confidence Interval Upper
Dummy Africa	0.6308945	.0854698	7.38	0.00	.4633769	.7984122
Dummy Latin America Caribbean	-.478894	.0814781	-5.88	0.00	-.6385881	-.3192
Dummy Eastern Europe	-.554775	.0820454	-6.76	0.00	-.715581	-.3939689
Dummy Western Europe	-1.117505	.0812616	-13.75	0.00	-1.276775	-.9582354
Population Share	12.97568	.7934082	16.35	0.00	11.42062	14.53073
Distribution Weight	.0000975	4.12e-06	23.68	0.00	.0000894	.0001055
Constant (Africa)	-5.374323	.0840844	-63.92	0.00	-5.539125	-5.20952

Note: Fractional logit regression by generalized linear model (glm) with a logit link and the binomial family. Robust standard errors clustered on each region (5 clusters). Intercept is Asia-Pacific Group. Log pseudolikelihood = - -4.75818526, AIC = 0 .0929124, BIC = -838.6563. Number of observations = 167, residual degrees of freedom = 164. Deviance =0 .6947290693, Pearson = 0. .7617015233, scale parameter = 1.

Table A4.4. Marginal Impacts for Geometric Mean Regression Results for Impacts of Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|
Dummy Africa	.0037192	.0004706	7.90	0.00
Dummy Latin America Caribbean	-.0028232	.0005061	-5.58	0.00
Dummy Eastern Europe	-.0032705	.0005135	-6.37	0.00
Dummy Western Europe	-.0065879	.0005394	-12.21	0.00
Population Share	.0764941	.0040513	18.88	0.00
Distribution Weight	5.75e-07	2.78e-08	20.71	0.00

Table A4.5. Original Formula with Floor and Ceiling Regression Results for Impacts of ISA Regional Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|	95% Confidence Interval Lower	95% Confidence Interval Upper
Dummy Africa	.2643064	.2023968	1.31	0.192	-.132384	.6609968
Dummy Latin America Caribbean	-1.417124	.105348	-13.45	0.00	-1.623602	-1.210645
Dummy Eastern Europe	-1.732336	.1033668	-16.76	0.00	-1.934932	-1.529741
Dummy Western Europe	-2.946661	.0973595	-30.27	0.00	-3.137482	-2.75584
Population Share	16.73391	.8501719	19.68	0.00	15.0676	18.40021
Distribution Weight	.0238255	.0032866	7.25	0.00	.0173839	.0302672
Constant (Africa)	-5.503984	.0995008	-55.32	0.00	-5.699002	-5.308966

Note: Fractional logit regression by generalized linear model (glm) with a logit link and the binomial family. Robust standard errors clustered on each region (5 clusters). Intercept is Asia-Pacific Group. Log pseudolikelihood = -4.112741492, AIC = .0971586, BIC = -833.0618. Number of observations = 167, residual degrees of freedom = 164. Deviance = 1.171170022, Pearson = 1.583409055, scale parameter = 1.

Table A4.6. Marginal Impacts for Original Formula with Floor and Ceiling Regression Results for Impacts of ISA Regional Groups, Distribution Weight, and Share of Global Population on Article 140 Allocated Shares ($\eta = 1$)

Variable	Coefficient	Robust Standard Error	z	P>|z|
Dummy Africa	.001507	.0011408	1.32	0.186
Dummy Latin America Caribbean	-.0080803	.0006877	-11.75	0.00
Dummy Eastern Europe	-.0098776	.0006968	-14.18	0.00
Dummy Western Europe	-.0168015	0007420	-22.64	0.00
Population Share	.0954148	.0039885	23.92	0.00
Distribution Weight	.0001359	.0000197	6.91	0.00

The following tables depict correlation coefficients among allocated shares S_i, population share P_i, social distribution weight ω_i, and per capita GNI, GNI_i, with $\eta = 1$. The social distribution weight and population share is always positively and statistically significantly correlated with the allocated share and the correlation is always higher for population share compared to social distribution weight. Per capita Gross National Income is negatively correlated with the allocated shares and is statistically significantly correlated with the geometric mean and original floor and ceiling formulae but is not statistically significantly correlated with the original formula. Population share and social distribution weight is not statistically significantly correlated for all three formulae.

Table A4.7. Correlation Coefficients for Allocated Shares, Population Share, Social Distribution Weight, and GNI $\eta = 1$: Original Formula

	Allocated Share S_i	Per Capita Gross National Income GNI_i	Social Distribution Weight ω_i	Population Share P_i
Allocated Share S_i	1.0000			
Per Capita Gross National Income GNI_i	-0.1289 (0.0970)	1.0000		
Social Distribution Weight ω_i	0.2190 (0.0045)	-0.2997 (0.0001)	1.0000	
Population Share P_i	0.7911 (0.0000)	-0.0665 (0.3930)	-0.0223 (0.9768)	1.0000

Note: Standard errors in parentheses.

Table A4.8. Correlation Coefficients for Allocated Shares, Population Share, Social Distribution Weight, and GNI $\eta = 1$: Geometric Mean Formula

	Allocated Share S_i	Per Capita Gross National Income GNI_i	Social Distribution Weight ω_i	Population Share P_i
Allocated Share S_i	1.0000			
Per Capita Gross National Income GNI_i	-0.2879 (0.0002)	1.0000		
Social Distribution Weight ω_i	0.4674 (0.0000)	-0.2997 (0.0001)	1.0000	
Population Share P_i	0.7156 (0.0000)	-0.0665 (0.3930)	-0.0023 (0.9768)	1.0000

Note: Standard errors in parentheses.

Table A4.9. Correlation Coefficients for Allocated Shares, Population Share, Social Distribution Weight, and GNI $\eta = 1$: Original Floor and Ceiling Formula

	Allocated Share S_i	Per Capita Gross National Income GNI_i	Social Distribution Weight ω_i	Population Share P_i
Allocated Share S_i	1.0000			
Per Capita Gross National Income GNI_i	-0.1877 (0.0151)	1.0000		
Social Distribution Weight ω_i	0.3720 (0.0000)	-0.2997 (0.0001)	1.0000	
Population Share P_i	0.7929 (0.0000)	-0.0665 (0.3930)	-0.0023 (0.9768)	1.0000

Note: Standard errors in parentheses.

APPENDIX 5. Inequality Measures

A number of inequality measures exist. The Atkinson inequality measure makes inequality judgments and derives measures from social welfare functions, giving a normative basis. The Generalized Entropy (Theil) inequality measures approach the quantification of inequality through comparing probability distributions and an information theory, although it can be lined to social welfare functions. The Gini coefficient (and Lorenz curve) can also be linked, under certain conditions, to the social welfare function. Thus, the three inequality measures, Atkinson, Generalized Entropy (Theil), and Gini coefficient, not only measure relative inequality but they also provide normative judgments in terms of which allocation formula for S_i provides the highest social welfare for the States Parties to the ISA.

ATKINSON $\boldsymbol{\gamma} \neq 1$: $A_\gamma = 1 - \left[\frac{1}{N}\sum_{i=1}^{N}\left[\frac{S_i}{\bar{S}}\right]^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, $0 \leq A_\gamma \leq 1$, smaller $A_\gamma\ more\ equal$

ATKINSON $\boldsymbol{\gamma}= 1$: $A_\gamma = \frac{\prod_{i=1}^{N} GNI_i^{\frac{1}{N}}}{\sum_{i=1}^{N} GNI_i}$, , $0 \leq A_\gamma \leq 1$, smaller $A_\gamma\ more\ equal$

Generalized Entropy 1 or THEIL T: $GE(1) = \frac{1}{N}\sum_{i=1}^{N}\frac{S_i}{\bar{S}}\, ln\left[\frac{S_i}{\bar{S}}\right]$, $0 \leq GE(1) \leq \infty$

- Smaller values are more equal
- More sensitive to distribution of shares in higher range than Theil L.

Generalized Entropy 0 or THEIL 0: $GE(0) = \frac{1}{N}\sum_{i=1}^{N} ln\left[\frac{\bar{S}}{S}\right]$, $0 \leq GE(0) \leq \infty$

- Smaller values are more equal
- More sensitive to distribution of shares in lower range than Theil T.

GINI COEFFICIENT: $\frac{1}{2N^2\bar{S}}\sum_{i=1}^{N}\sum_{j=1}^{N} S_i - S_j,\ i \neq j, 0\ \leq G \leq 1$

- Smaller values are more equal
- Lorenz Curve is graphical representation of Gini

Atkinson inequality index values can be used to calculate the proportion of total income that would be required to achieve an equal level of social welfare as at present if incomes were perfectly distributed. For example, an Atkinson index value of 0.20 suggests that we could achieve the same level of social welfare with only 1–0.20 = 80% of income. The theoretical range of Atkinson values is 0 to 1, with 0 being a state of equal distribution. The Atkinson index incorporates a sensitivity parameter (γ). This parameter γ can range from 0 (meaning that the ISA is indifferent about the nature of the income distribution), to infinity (where the ISA is concerned only with the income position of the very lowest income group), i.e. $0\ \leq\ \gamma\ \leq 1$. Atkinson argued that this index was a way to incorporate Rawls' conception of social justice into the measurement of income inequality. In practice, γ values of 0.5, 1, 1.5 or 2 are used; the higher the value of γ, the more sensitive the Atkinson index becomes to inequalities at the bottom of the income distribution.

The Generalized Entropy (Theil) inequality index measures an entropic "distance" the population is away from the egalitarian state of everyone having the same income. The numerical result is in terms of negative entropy so that a higher number indicates more order that is further away from the complete equality. For lower values of α, the measure is more sensitive to changes in the lower tail of the distribution and, for higher values, it is more sensitive to changes that affect the upper tail (Atkinson and Bourguignon, 2015). The most common values for α are 0, 1, and 2. The more positive α (the sensitivity parameter; −1, 0, 1 or 2) is, the more sensitive $GE(\alpha)$ is to inequalities at the top of the income distribution. The theoretical range of $GE(\alpha)$ values is 0 to infinity, with 0 being a state of equal distribution and values greater than 0 representing increasing levels of inequality, i.e. $0 \leq GE(\alpha) \leq \infty$. Another beneficial property of the $GE(\alpha)$ measure is that it is decomposable; that is, it can be broken down to component parts (i.e. population subgroups). This enables analysis of between- and within-area effects

The Gini coefficient is a relative inequality measure largely associated with the descriptive approach to relative inequality measurement, although it can be linked to social welfare functions and social welfare analysis. The Gini coefficient attempts to distill a two-dimensional area (the gap between the Lorenz curve and the equality line) down in to a single number, it obscures information about the "shape" of inequality. In particular, the Gini coefficient measures the ratio of the area between the Lorenz curve and the equi-distribution line to the area of maximum concentration. The generalized Gini coefficient is dependent upon the degree of relative inequality aversion, but neither the generalized Gini coefficient or Gini coefficient as the primary social welfare measure are not developed or used here. Instead, the report instead uses the Atkinson and Generalized Entropy (Theil) inequality measures with different values of inequality aversion to evaluate social welfare impacts of alternative allocations S_i.

More on Atkinson Index. In the words of Atkinson (1970), A_γ is 1 minus the ratio of the equally distributed equivalent level of income to the mean of the actual distribution. If A_γ falls, then the distribution has become more equal-we would require a higher level of equally distributed income (relative to the mean) to achieve the same level of social welfare as the actual distribution. The measure A_γ has the convenient property of lying between 0 (complete equality) and 1 (complete inequality). Moreover, this new measure has considerable intuitive appeal. If A_γ = 0.3, for example, it allows us to say that if incomes were equally distributed, then we should need only 70% of the present national income to achieve the same level of social welfare (according to the particular social welfare function). Or we could say that a certain plan for redistributing income would raise social welfare by an amount equivalent to an increase of 5 % in equally distributed income. This facilitates comparison of the gains from redistribution with the costs that it might impose-such as any disincentive effect of income taxation-and with the benefits from alter- native economic measures.

Given any income distribution, therefore, GNI_{EDE} can be easily calculated for different levels of inequality aversion. Different levels of inequality aversion γ give different values of GNI_{EDE}. For $\gamma = 0$, the equally distributed equivalent income is simply the average level of income. With $\gamma > 0$, GNI_{EDE} decreases (for convex social welfare function, its level is always below average income) and A_γ increases. For example, if with $\gamma = 2$=, $A_\gamma(\gamma = 2)$ = 0.379, the interpretation is that society is disposed to release 37.9 per cent of the size of the cake to have equal slices. If $\gamma \to \infty$. the Rawlsian criterion is used, i.e. the social welfare function becomes more and more inequality averse.

The Atkinson Index is predicated upon an iso-elastic social welfare function: : $W(U) = \left[\frac{1}{1-\gamma}\sum_{i=1}^{N}[(U_i)^{1-\gamma}]\right]$. When the SWF is a direct function of income, Y_i, the SWF is written: $W(Y) = \left[\frac{1}{1-\gamma}\sum_{i=1}^{N}[(Y_i)^{1-\gamma}]\right], \gamma \neq 1$ When $\gamma = 1, W(Y) = logY_i$. For both SWFs, the relative inequality aversion parameter γ is a constant and $0 \leq \gamma \leq \infty$ with quasi-concavity of the SWF (strict concavity gives <

instead of $\leq$). The term $\frac{1}{1-\gamma}$ ensures that U_i rises with income, no matter whether γ is above or below unity. The coefficient of relative inequality aversion is: $\gamma = -U_i \frac{\frac{\partial^2 W(U)}{\partial U_i^2}}{\frac{\partial W(U)}{\partial U_i}}$.

Different values of the relative inequality aversion parameter γ give different SWFs. When $\gamma = 0, W = \sum_{1=1}^{N} U_i$ or $= \sum_{1=1}^{N} Y_i$, i.e. the utilitarian SWF. When $\gamma \rightarrow \infty, W = min(U_1, U_2, \ldots, U_N)$ or $W = min(Y_1, Y, \ldots, Y_N)$ i.e. the $min(Y_1, Y, \ldots, Y_N)$ i.e. the Rawlsian SWF. When $\gamma \rightarrow 1, W = \prod_{i=1}^{N} U_i$ or $W = \prod_{i=1}^{N} Y_i$, i.e. the Bernoulli-Nash (Cobb-Douglas) SWF for total utility (or $\left[\prod_{i=1}^{N} U_i\right]^{\frac{1}{N}}$ or $\left[\prod_{i=1}^{N} Y_i\right]^{\frac{1}{N}}$ the geometric mean for average utility or income). When $\gamma = 1$, $W = \sum_{i=1}^{N} lnU_i$ or $W = \sum_{i=1}^{N} lnY_i$, the SWF treats equal proportional increases in utility, income or consumption equally across countries/individuals. That is, the SWF $W = \sum_{i=1}^{N} lnU_i$ treats an X% increase for the poorer country the same as for a better-off country. When $\gamma > 1$, the SWF treats an x% increase for the poorer country as more welfare increasing than x% for the better-off country. As γ increases toward infinity (the Rawls SWF), small increases in income or utility for the worst-off get weighted much more than large increases in income or utility. In the limit, the Rawlsian case, increases in income or utility for the better-off do not impact welfare.

The parameter γ, is as noted above, $\gamma = -U \frac{\partial^2 W(U)/\partial U^2}{[\partial W(U)/\partial U]}$, where $\partial^2 W(U)/\partial U^2$ reflects the rate at which the marginal social utility declines with higher levels of utility and $\partial W(U)/\partial U$ is marginal social utility or the change in social welfare with a change in utility. The parameter γ indicates the amount by which welfare declines with an increase in income, i.e. the relative inequality aversion. The higher is γ, the higher is the relative aversion to inequality in utilities. The higher is γ, the faster is the rate of proportional decline in welfare to a proportional increase in income (or utility). γ captures the extent to which the social planner wants to place higher values on monetary gains accruing to various countries, i.e. the inequality-aversion coefficient captures the moral/ethical principles of a social planner who prefers to give some priority to utility changes affecting worse-off countries rather than simply aggregating utilities in a utilitarian manner.

The relative inequality aversion parameter γ is related to the elasticity for the social marginal welfare of income $\eta = \varepsilon + \gamma(1 - \epsilon)$. η is a function of two parameters, the coefficient of relative inequality aversion γ and elasticity of the private marginal utility of income (or consumption) ε. The elasticity for the social marginal welfare of income, which is assumed to be the same for every country's income, is defined as: $\eta = -Y_i \frac{\frac{\partial^2 W(Y)}{\partial Y_i{}^2}}{\frac{\partial W(Y)}{\partial Y_i}}$.[1] The elasticity for the social marginal welfare of income η indicates the concavity of the composite function $W(Y)$ with respect to its argument, income Y and the overall social preference for income redistribution. η contains both the relative risk aversion parameter ε and relative inequality aversion parameter (both intra- and inter-generationally) γ, and is thus a mixture of risk aversion and ethical values. However, usually only one parameter is used, intra-generational distribution, so that relative risk aversion and inter-generational income distribution are implicitly held constant.

Atkinson's inequality index is predicated upon the concept of Equally Distributed Equivalent (EDE) income. EDE is that level of income that, if obtained by every individual in the income distribution, would enable the society to reach the same level of welfare as actual incomes.

The equally distributed equivalent level of income GNI_{EDE} is the level of per capita income which if equally distributed would give the same level of social welfare as the present distribution. The Atkinson Inequality Index A_γ is then defined as:

$$A_\gamma = 1 - \frac{GNI_{EDE}}{\overline{GNI}},$$

where $GNI_{EDE} = \left[\frac{1}{N}\sum_{i=1}^{N} GNI_i^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$ and $\overline{GNI} = \sum_{i=1}^{N} GNI_i$, i.e. the arithmetic mean of GNI.

Using the explicit formula for the iso-elastic social welfare function and substituting the terms just defined gives:

$$A_\gamma = 1 - \left[\frac{1}{N}\sum_{i=1}^{N}\left[\frac{GNI_i}{\overline{GNI}}\right]^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

When $\gamma = 1$, $GNI_{EDE} = \prod_{i=1}^{N} GNI_i^{\frac{1}{N}}$, i.e. the geometric mean of GNI. When $\gamma = 1$, then

$$A_\gamma = \frac{\prod_{i=1}^{N} GNI_i^{\frac{1}{N}}}{\sum_{i=1}^{N} GNI_i},$$

or in words, the ratio of the geometric mean of GNI to the arithmetic mean of GNI.[17]

Reference:

Cowell, Frank A. 2009. Measuring Inequality. Available at (accessed 23 January 2020): http://darp.lse.ac.uk/papersdb/Cowell_measuringinequality3.pdf

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[17] In the words of Atkinson (1970), A_γ is 1 minus the ratio of the equally distributed equivalent level of income to the mean of the actual distribution. If A_γ falls, then the distribution has become more equal-we would require a higher level of equally distributed income (relative to the mean) to achieve the same level of social welfare as the actual distribution. The measure A_γ has the convenient property of lying between 0 (complete equality) and 1 (complete inequality). Moreover, this new measure has considerable intuitive appeal. If A_γ = 0.3, for example, it allows us to say that if incomes were equally distributed, then we should need only 70% of the present national income to achieve the same level of social welfare (according to the particular social welfare function). Or we could say that a certain plan for redistributing income would raise social welfare by an amount equivalent to an increase of 5 % in equally distributed income. This facilitates comparison of the gains from redistribution with the costs that it might impose-such as any disincentive effect of income taxation-and with the benefits from alter-native economic measures. Given any income distribution, therefore, GNI_{EDE} can be easily calculated for different levels of inequality aversion. Different levels of inequality aversion γ give different values of GNI_{EDE}. For $\gamma = 0$, the equally distributed equivalent income is simply the average level of income. With $\gamma > 0$, GNI_{EDE} decreases (for convex social welfare function, its level is always below average income) and A_γ increases. For example, if with $\gamma = 2$=, $A_\gamma(\gamma = 2)$ = 0.379, the interpretation is that society is disposed to release 37.9 per cent of the size of the cake to have equal slices. If $\gamma \to \infty$. the Rawlsian criterion is used, i.e. the social welfare function becomes more and more inequality averse.

Easily accessible and read discussions of inequality measures include:

Bellù, L.G. and P. Liberati. 2005. Charting Income Inequality: The Lorenz Curve. Rome: Food and Agriculture Organization of the United Nations. Available at (accessed January 23 2020): http://www.fao.org/docs/up/easypol/302/charting_income_inequality_000en.pdf

Bellù, L.G. and P. Liberati. 2005. Social Welfare Analysis of Income Distributions: Ranking Income Distributions with Lorenz Curves. Rome: Food and Agriculture Organization of the United Nations. Available at (accessed January 23 2020): http://www.fao.org/3/a-am390e.pdf

Bellù, L.G. and P. Liberati. 2006. Welfare Based Measures of Inequality: The Atkinson Index. Rome: Food and Agriculture Organization of the United Nations. Available at (accessed January 23 2020): http://www.fao.org/docs/up/easypol/451/welfare_measures_inequa_atkinson_050EN.pdf

Bellù, L.G. and P. Liberati. 2006. Inequality Analysis: The Gini Index. Rome: Food and Agriculture Organization of the United Nations. Available at (accessed January 23 2020): http://www.fao.org/docs/up/easypol/329/gini_index_040en.pdf

Bellù, L.G. and P. Liberati. 2006. Describing Income Inequality: Theil Index and Entropy Class Indexes. Rome: Food and Agriculture Organization of the United Nations. Available at (accessed January 23 2020): http://www.fao.org/3/a-am343e.pdf

Appendix 6. Equitable Sharing and Social Distribution Weights

Equity is a complex idea that resists simple formulations. It is strongly shaped by cultural values, by precedent, and by the specific types of goods and burdens being distributed. To understand what equity means in a situation we must therefore look at the contextual details. Equity is a central concern in the most basic political decisions. All these distributive problems can be and are solved without invoking theories of social justice. It is possible to analyze the meaning of equity in the small without resolving what social justice means in the large.

As the ISA grapples with this challenge, several issues will need to be addressed. These include: (i) the principles to be used in determining the "claims" that different entities or groups (current or future) will have on the pool of resources that are generated, and (ii) the mechanisms to be used for distributing available funds, including whether distribution should be in the form of direct payments to States or funded projects. The first issue is, essentially, a question of what equitable sharing means in the context of DSM. The second is a question about how equitable sharing can or should be achieved.

As a general principle, the equitable sharing of resource rents can be based on two possible rationales. The first is simply based on the concept of shared ownership. Alternatively, equitable sharing can reflect an implicit or explicit desire to redistribute income or wealth, presumably from wealthier States to poorer States. In this case, shares should be distributed based on some indicator of a State's priority in the redistribution goal, and would, typically, embody some form of progressivity that favours poorer States in the distribution scheme.

Progressivity can be defined in various ways. For example, it can mean (i) that the share of rents received by a low-income State is higher than the share received by a high-income State, or (ii) that the total amount received as a percentage of income is higher for low income States than for high income States. The first definition is more favourable to low-income States,[18] but either implies a redistribution of income or wealth relative to what would be required by a proportional distribution scheme based solely on ownership rights.

Article 140 provides that DSM must be carried out for the benefit of mankind as a whole, irrespective of the geographical location of States, whether coastal or landlocked. This implies a joint ownership rationale for equitable sharing. Article 140 also requires the ISA to take into particular consideration the interests and needs of developing States and of peoples who have not attained full independence or other self-governing status, implying an income redistribution rationale as well.

Beyond establishing basic principles for implementing the somewhat ambiguous guidance regarding the target beneficiaries discussed above, the ISA will also need to develop more specific principles and associated metrics for conceptualizing a hierarchy of needs and equitable shares. Metrics based on population and per capita income, such as those used to determine United Nations budget

[18] Technically, if s denotes the share of some fixed amount of revenue R that is distributed to a State with income of Y, then the first definition of progressivity requires that sR increases as Y decreases, which requires that s be inversely related to income. In contrast, the second definition requires that sR/Y increases as Y decreases. This can hold even if shares are the same or even increasing in income, i.e., higher income States receive a greater share, as long as the percentage difference in the share is less than the percentage difference in income.

contributions, could be used.[19] Alternatively, priorities could be based on a composite index combining various well-known and generally accepted development indicators and statistics. These might include, for example, the Human Development Index maintained by the United Nations Development Programme and the World Development Indicators developed by the World Bank.

Rather than developing and discussing the philosophical, ethical, and semantic meaning of equitable, fair, or just distributions in the large, as discussed above this report concentrates upon equitable sharing in the small. This sharing follows the LOSC as well as the procedures and norms of the ISA and largely focuses upon the more practical discussions of how the equitable sharing of DSM royalties, received by the ISA, will be defined, measured, and implemented in the small.

Equity (in the small) could be defined as a state in which each ISA member's welfare is increased to the maximum extent possible, without making any other ISA member worse off, given the limited resources in the Area and the deep seabed mining returns available for distribution, after taking proper account of the LOSC, the common heritage of mankind principle, and the sharing rules the ISA considers appropriate to its need. Alternatives are possible, such as for example, equity with a stability property called "no justifiable envy". Equitable sharing has justifiable envy if a State Party would prefer another allocation to that which it receives when a State Party of higher income receives a larger allocation of proceeds.

Equitable sharing in this report refers to sharing rules that the ISA considers appropriate to its needs rather than an abstract moral or ethical construct in the large. Appropriateness is shaped partly by principle, partly by precedent, and partly by what can be practically implemented. Appropriateness expresses what is reasonable and customary in a sharing situation. Appropriateness can be subjective through the stated preferences of the ISA States Parties or based upon the revealed preferences of policy makers.

The analysis uses the revealed preference of the highest possible global authority and representation of humanity, the UN General Assembly, to develop appropriateness and income progressivity as implied by the UN General Assembly's formula for assessed contributions in a manner consistent with the LOSC.[20] This revealed preference is embodied in social distribution weights and the subsequent equitable allocation shares for States Parties. This revealed preference, based upon decisions made independently of the allocation problem at hand, may come close to being "strategy proof" to the extent that each UN General Assembly member (with its own private information) honestly reveals its preferences on global progressivity in an action unrelated to the progressive distribution of the proceeds from deep seabed mining to the States Parties of the ISA.

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[19] See United Nations, 'Possible Methods and Criteria for the Sharing by the International Community of Proceeds and Other Benefits Derived from the Exploitation of the Resources of the Area Beyond the Limits of National Jurisdiction,' United Nations General Assembly, Committee on the Peaceful Uses of the Sea-bed and the Ocean Floor Beyond the Limits of National Jurisdiction, A/AC.138/38, mimeo, 1971, for some illustrative examples of ways for defining shares based on population but adjusted for per capita income (to favour developing countries).

[20] Dietz et al. (2008, pages 7-8) observe that, "To deduce ethical values from preferences revealed by behaviour, at least four (non-trivial) conditions would be required: (i) a unique preference is revealed by the observed behaviour (the 'inverse optimum problem'); (ii) the preferences revealed are the 'true preferences' of the individual, based on full and correct information without any errors in decision-making; (iii) the preferences measured are contextually relevant to the ethical judgement at hand; and (iv) the preferences are appropriate for social decision making, and not merely individual decision making." Dietz et al. discuss these issues and problems with using market data to established revealed preferences in favor of stated preferences (and this approach's issues).

Aristotle's equity principle or proportionality principle states that the goods or services of concern should be divided in proportion to each's claimant's contribution (or claim). This approach requires that the good must be divisible and requires measuring each claimant's contribution (claim) on a cardinal scale that can be expressed in a common metric, which is sometimes clear and in other case is not. When entitlement is created by verifiable and fungible claims, the proportional rule has the advantage through treating the units of claim equally, rather than the States Parties which possess them. A division of resources in equal shares for all participants is non-envious, but it is generally inefficient. The fair share guarantee states that a States Party should not strictly prefer the proportional share to the actual allocation and is an ex ante lower bound on individual welfares in the sense that fair share does not depend on the preferences of States Parties other than another States Party. Aristotle's equity principle can have an incentive effect through overbidding (parties claim for more shares than they really want).[21]

The theory of cooperative games has developed a theory of allocations. Some of the approaches may or may not face difficulty in practical operationalization, others may be more tractable.[22] An allocation rule based on the "Contested Garment Principle" lies in the nucleolus.[23] In sum, allocations based upon cooperative solutions from game theory (along with allocations based upon welfare economics developed below) deserve attention.

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[21] Moulon, F. 2003. *Fair Division and Collective Welfare*, M.I.T. Press, Cambridge. Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

[22] A Shapley value is a systematic formula used to divide a joint cost or a jointly produced output. It offers a reasonable definition and computation of the share of cost or surplus for which a user of the commons is deemed responsible. The Shapley value essentially weights States Parties based upon their marginal contributions, such that each States Party contributes the more they stand to gain. The Shapley value can yield inconsistent answers. The nucleolus is a unique solution that maximizes the benefits to the least-satisfied coalition, and is thus comparable to the minimax principle of Rawls (1971). The Nash bargaining solution, an egalitarian approach, essentially assumes that all States Parties in a coalition are equally important because full cooperation would not succeed without all of them, and thus the payoff should be shared equally. The Nash bargaining solution is thus closely related to Aristotle's equity principle.

[23] The rule is based on allocation of the contested claim and meets the properties of the nucleolus. The claims problem is as follows. Several individuals (States Parties) have claims on a common asset, and the claims exceed the amount available. (Here the asset is perfectly divisible.) A solution to the claims problem is the division of the total amount among the various claimants, such that no individual receives more than that individual's claim and no claim is zero. Two claims are important to distinguish: (1) voluntary claims, when claims are created by voluntary actions, in which case the incentive impact can be important, and (2) involuntary claims, which involve no choice or effort on the part of the claimant, in which case the incentive impact may not be important. The Contested Garment Rule is then: Let two individuals have claims against a common asset, where the sum of the claims exceeds (or equal to) the total amount. Each claimant's uncontested portion is the amount left over after the other claimant has been paid in full in case that claim is less than the total, and zero otherwise. The contested garment rule gives each claimant his/her uncontested portion plus one-half of the excess over and above the sum of the uncontested portions. General Rule: (1) Equal amounts if the total is less than the smallest claim; (2) Equal loss if the total is more than the largest claim; (3) Half of the individual's claim to smallest claimant and rest to other in all other situations. An allocation among a group of claimants is pairwise consistent with the contest garment rule if every two claimants share the total allocated to them according to the contested garment rule. Another rule is Maimonides's Rule, in which an equal amount is given to each claimant or the full amount of each individual's claim, whichever is smaller. See Chapter 4 of P. Young. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

A number of allocation options are simply not tractable for various reasons. They may not be fair, efficient, or homogeneous and divisible, or not be practicably applicable to the equitable distribution problem at hand. Auctions, divide-and-choose, lotteries, rotation, queuing, or profit sharing cannot be practically operationalized. Auctions would likely be considered unfair by lower income States Parties (since auctions rely upon the ability to pay, which favors higher income States Parties), and in any case, lead to inequitable outcomes when the States Parties have substantially different private endowments. Divide-and-choose, queuing, and lotteries could be used in allocating contracts. Rotation (time sharing) of mining opportunities according to some rule presents another alternative but is inefficient (at a minimum) and not applicable to mining an exhaustible resource with large irreversible investments or to equitable division of the mining royalties. Divide-and-choose, lotteries, rotations, and queuing (likely first come, first serve) fail to satisfy some notion of priority among claimants.[24] Priority methods of allocation are the only ones that allocate a good impartially and consistently over different situations, even though the criteria upon which priority is based may differ greatly from one situation to another.[25] Appendix 6 discusses priority methods within the context of developing weights for aggregating different criteria important in developing allocation formulae.

An allocation is a fair division when claimants decide directly rather than a third party. Hence, voluntary and self-enforcing international bodies and negotiations within these bodies will make the equitable allocation a fair division. From a procedural point of view the decision must be unanimous. We shall also assume that the good is divisible, or if it is not, that they divide chances at getting the good.

Strictly speaking, the distribution is not an envy-free distribution, since envy-free distribution only applies when parties have equal claims on the divisible good (here DSM royalties), which is not strictly true due to the priorities outlined in LOSC Article 140 (and Article 82). Because most equitable and fair division problems resolve around the question of how differences in claims (due to disparities in merit, contribution, need, etc.) should be considered, and to the extent that the degree of progressivity as agreed upon by the ISA States Parties in principle resolves differences in claims, the issue of envy-free equitable royalty distribution should become of little or no relevance

An allocation is envy-free if no individual prefers another’s portion to one’s own (Mouton 2003). Envy-freeness as a principle for equity states that global society is not in general envy-free, but rather that no individual prefers another portion of a particular allocation of goods or services. An equitable sharing would be Pareto efficient if other feasible allocations do not make at least one individual better off without making at least one other individual worse off.

Envy-free distribution only applies when parties have equal claims on the good, which is not strictly true here due to the LOSC Articles 140 and 82 (even though the distribution starts with equal claims on the DSM proceeds due to the status of the resources as the common heritage of mankind and per capita distribution, giving Aristotle’s proportionality principle, as the initial basis of the distribution).

Because the equitable sharing allocation is made after, and independent of, the decision for the amount and type of contractor payment to the ISA, there is not an equity-efficiency trade-off in this dimension,

[24] Divide-and-choose was operational for deep seabed mining claims because the Area was owned by humanity as a whole under the common heritage of mankind concept and deep seabed mining had no history of actual mining. Divide and choose is envy-free.
[25] Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

including no impact upon the supply of contractor effort. Inter-generational equitable sharing, as discussed previously, can be addressed through measures such as the Seabed Sustainability Fund (which could also be considered as a Common Heritage Fund).[26]

Social Distribution Weights. As discussed in the original report, social distribution weights $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^\eta$ indicate the marginal social value of an extra unit of income to individual States Party i. They represent the value that society places on providing an additional dollar of income or consumption to any given individual. These weights directly reflect society's concerns for fairness. The weight attached to each States Party i when it receives an extra unit of income is positive, and the more income a States Party receives the smaller the relative social weight becomes.

The ratio $\frac{\overline{GNI}}{GNI_i}$ makes the social distribution weight $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^\eta$ inversely proportional to each States Party's per capita GNI relative to all States Parties' per capita GNI, i.e. $\frac{\overline{GNI}}{GNI_i}$, and thus makes the distribution progressive. The weight ω_i has a value of one at $\overline{GNI}$. When $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} > 1$, greater weight is given to a States Party's population share P_i and hence the allocation of a States Party i whose per capital GNI is less than the mean per capita GNI of all States Parties (i.e. $GNI_i < \overline{GNI}$). When $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta=1} < 1$, greater weight is given to P_i and hence the allocation of a States Party i whose per capita GNI is greater than the mean per capita GNI of all States Parties. The parameter value η further contributes to or reduces the progressivity, where a value $\eta < 1$ reduces progressivity and a value $\eta > 1$ increases progressivity, and a value $\eta = 0$ makes all social distribution weights equal and unity. The parameter value $\eta > 1$ gives greater weight to beneficiaries the lower their per capita GNI relative to mean global per capita GNI, $\overline{GNI}$.

The Elasticity of the Social Marginal Welfare of Income, $\boldsymbol{\eta}$, captures the change in social welfare for an increase in an individual's income due to the decline in social weight as utility increases and the declining marginal utility of income as income increases. In this exercise, η can also be interpreted as a measure of global society's aversion to inequality. The value for η incorporates attitudes to risk, inequality within generations, and inequality between generations. Higher η implies: (1) greater risk aversion, and/or (2) greater social inequality aversion (increasing the relative weight placed on changes in the consumption or income of the lower income and less weight to the higher income, increasing the overall gain in social welfare), and/or (3) if we also assume, as is standard, that aggregate consumption and income will continue to grow, then the overall gain in social welfare increases with higher η because it reduces the weight placed on future consumption, and increases the weight placed on present consumption (because the present is poorer than the future). Similarly, lower η implies less social inequality aversion (decreasing relative weight on income for lower income and increasing relative weight on income for higher income) or greater weight on future compared to present consumption. Spreading risk among all States Parties minimizes any residual risk. Values of η corresponds to different ethical values, where $\eta = 0$ gives Utilitarianism (the linear utility function which ranks distributions

[26] During UNCLOS III, the establishment of a Common Heritage Fund was proposed by a group of 9 States (Afghanistan, Austria, Bolivia, Lesotho, Nepal, Singapore, Uganda, Upper Volta (Burkina Faso) and Zambia), but in relation to funds received pursuant to article 82, and not in connection with funds paid for activities in the Area. The Group for the Common Heritage Fund advanced the proposal in the Second Committee in '… an effort to strengthen those provisions of the Law of the Sea draft treaty which try to implement the vision of the oceans beyond national jurisdiction as the common heritage of mankind.' See Virginia Commentary, Vol. II at 945. Also, NG6/13 (1979, mimeo.). Reproduced in IX Platzöder 383.

solely according to income) and $\eta = \infty$ corresponds to the maximin principle. Utilitarianism, which states that the goods or services of concern should be distributed to maximize the total welfare of the claimants – the greatest good for the greatest number. Utilitarianism also can impose harm on a few to confer a small benefit on the many. The maximin principle, due to Rawls (1971), has the central distributive principle that the least well-off in society or a group should be made as well-off as possible. Well-off does not pertain to an individual's subjective level of satisfaction, but instead to the means or instruments by which satisfaction can be achieve. Rawls J., 1972. *A Theory of Justice*, Oxford University Press, Oxford, UK.

The parameter η is derived from the UN General Assembly's revealed preferences by treating the UN assessment and subsequent contribution to budget as a progressive tax scheme (see Annex 4). The implicit marginal and average tax rates can be determined from these assessments. Each member's GNI is the income and each member's contribution is the tax, $T(GNI)$. These values are converted to per capita values for each States Party. The marginal tax rate is $\frac{\partial T(GNI)}{\partial GNI}$, where the symbol ∂ denotes the first partial derivative, and the average tax rate is $\frac{T(GNI)}{GNI}$. These tax rates enter into a formula that gives the elasticity of the marginal utility of income $\eta = \frac{1-ln(MTR)}{1-ln(ATR)}$, which in turn enters into the above formula that gives distribution weights $\omega_i = \left[\frac{\overline{GNI}}{GNI_i}\right]^{\eta}$.

This report assumes that the ISA has already made the intra-generational and inter-generational decision (so that reason #3 is irrelevant) and that there is minimal or no risk to the ISA States Parties since the distribution of royalties is separate and follows mining and that price and revenue volatility are addressed in a previous step (so that reason #1 plays a minimal role). Spreading risk among all States Parties also minimizes any residual risk. Annex 4 provides further discussion. Annexes 3 and 5 of the original report extensively discuss social distribution weights.

The social distribution weights ω_i are derived from the social welfare function of economics (as discussed in the original report) and from the UN's revealed preference for progressivity as indicated by the value of η, which was found to be $\eta = 1$ for Article 140 distributions

Social Welfare Functions. As discussed in the original report, the social welfare function of economics represents some ethical judgement about the appropriate distribution of social welfare across people affected by a policy change, here the distribution of proceeds (see Annex 5 of the original report for more extensive and formal development). It allows quantitative evaluations of outcomes to determine whether social welfare increases. A social welfare function cannot be observed. The social welfare function is normative and must instead be specified according to a particular ethical view. This social welfare function approach has the advantage of quantification according to well-developed principles of economics that captures ethics and allows either revealed or stated preferences for quantifying the ethical view that is assumed through the form of the social welfare function. The social welfare function approach is a consequentialist moral theory. As such, it says that policies should be judged only in terms of their consequences, and the only relevant consequences are to individual well-being. The social welfare function used in both this and the original report is a constant elasticity iso-elastic function that values both equality and high total social welfare (utility), is common across States Parties, and is a function of real (inflation-free) per capita Gross National Income (GNI) of States Parties. The constant-elasticity iso-elastic social welfare function gives increasing priority to utility changes the lower the per capita GNI of a States Party.

Appendix 7. Geometric Means, Cobb-Douglas Aggregator Functions, and Consistent Aggregation

The numerator in the original formula is multiplicative, because P_i and ω_i are multiplied together. The original formula corresponds to a Cobb-Douglas aggregator function for the numerator for each States Party i: $f(Z_{ij}) = A_0 \prod_{j=1}^{M} Z_{ij}{}^{A_j}$, where $A_0 = 1$, $A_j = 1, \forall j \in M$, $Z_{ij} = P_i, \omega_i$, and here $M = 2$. (An aggregator function performs a calculation on a set of values, here P_i and ω_i, to return a single scalar value, here the numerator of S_i.) The resulting geometric index (here the numerator $f(Z_{ij})$) is exact for a Cobb-Douglas aggregator function. This aggregator function can be viewed as a first-order approximation to any arbitrary function in the neighborhood of initial values for P_i and ω_i. The key distinguishing factor in the original functional form is the linear exponent to P_i and ω_i (giving the Cobb-Douglas aggregator function is homogeneous of degree 2) where the geometric mean formula uses an exponent of $\frac{1}{M}$, i.e. $A_j = \frac{1}{M}$ and $\sum_{j=1}^{M} A_j = 1$ (so that the Cobb-Douglas aggregator function is homogeneous of degree 1). The original (multiplicative) formula treats equal proportional increases in P_i and ω_i equally across States Parties. The formula for the numerator of S_i roughly corresponds to the functional form of the Bermoulli-Nash social welfare function.

The geometric mean index decreases the level of substitutability between the dimensions [being compared], here P_i and ω_i, compared to the original geometric index (exponents of P_i and ω_i are 1). The Human Development Index uses geometric mean for this reason. Thus, a low value in one dimension is not linearly compensated by high achievement in another dimension. At the same time ensures that a 1 percent decline in S_i has the same impact on the allocation as a 1 percent decline in $\omega_i = \frac{\overline{GNI}}{GNI_i}^{\eta}$.

As a basis for comparisons of achievements, this method is also more respectful of the intrinsic differences across the dimensions than a simple arithmetic mean. The geometric mean is excellent for constructing composite indices, utilizing very different sorts of data that are all scored differently. The reason is, the geometric mean is indifferent to the scales used (as long as the same ones are used each time). The geometric mean, in contrast to an arithmetic mean, combines values with a product instead of a sum, and then split them up again with an Nth root. The conceptual difference is seeing each data point as a *scaling factor*, which combine by increasing each other multiplicatively. The geometric mean is what any scaling factor would be, if they were all the same. Moreover, the geometric mean is the only correct mean when averaging normalized results; that is, results that are presented as ratios to reference values such as S_i and ω_i. (Fleming, Philip J.; Wallace, John J. (1986). "How not to lie with statistics: the correct way to summarize benchmark results". *Communications of the ACM*. **29**(3): 218–221. doi:10.1145/5666.5673).

Thus, the geometric mean is also excellent for constructing composite indices, utilizing very different sorts of data that are all scored differently. The reason is, the geometric mean is indifferent to the scales used (as long as the same ones are used each time).

Other economic index numbers or aggregator functions exist, notably the quadratic-mean-of-order-r aggregator function with the corresponding superlative index, but they are not relevant in this case because two or more time periods or States Parties are not directly compared (in bilateral or multilateral) indices.

Consistency in Aggregation. The index C_i can be directly constructed in the allocation formula in a single step. The index C_i can also be constructed inn two or more stages. Multi-stage versus single-stage construction raises the issue of consistency in aggregation. An index-number formula is consistent in aggregation if the numerical value of the index constructed in two or more stages necessarily coincides with the value of the index calculated in a single stage (Vartia 1974). The geometric indexes, including those used in this study (which are essentially Cobb-Douglas aggregator functions) along with the Paasche and Laspeyres are consistent in aggregation (Vartia 1976). (Vartia, Y.O. 1974. *Relative Changes and Economic Indices*. Licentiate thesis, Univ. Helsinki. Vartia, Y.O. 1976. *Relative Changes and Index* Numbers. Research Institute of the Finnish Economy, Helsinki.) The superlative indices are not.

Consistent aggregation providing a perfectly satisfactory overall index that can be applied to individual periods in an intertemporal context, to individual economic entities, or to subgroups of commodities requires homothetic weak separability of the underlying aggregator function. Thus, to justify the two-stage method of calculating index numbers for any partition of variables requires an aggregator function, such as the Cobb-Douglas, which is homothetically separable in the same partition that corresponds to the two stages. The Paasche and Laspeyres indices are consistent in aggregation since the underlying aggregator function is either linear or Leontief, the Vartia I's underlying aggregator function is the Cobb-Douglas, and the Vartia II's underlying aggregator function is the CES. If the underlying aggregator function is not separable, any attempt to construct an overall or group, quantity index by using subgroup indices will result in the group-quantity index varying with variations in quantities of commodities outside of that group. An implicitly separable underlying aggregator function for an index also allows consistent aggregation. Blackorby et al. (1978). (Blackorby, C., D. Primont, and R.R. Russell. 1976. *Duality, Separability, and Functional Structure*. New York: North-Holland Publishing Company, 395 pp.)

APPENDIX 8. Stated and Revealed Approaches to Develop Appropriateness and Allocation Formulae Weights When Additional Criteria Are Added to Allocation Formula

Appropriateness in an equitable allocation can be subjective through the stated ethics of the CPCs or based upon the revealed ethics of policy makers. Stated ethics we discuss when we discuss weights for the individual allocation criteria.

The question arises of whose revealed ethics to use and how to define and measure these ethics. One source of revealed ethics is the highest possible global authority and representation of humanity, the UN General Assembly, to develop appropriateness and income progressivity as implied by the UN General Assembly's formula for assessed contributions in a manner consistent with the UNCLOS and UNFSA. This revealed ethics, based upon decisions made independently of the allocation problem at hand, may come close to being "strategy proof" to the extent that each UN General Assembly member (with its own private information) honestly reveals its ethics on global progressivity in an action unrelated to the progressive distribution of royalties by the ISA.

The revealed ethics can be embodied in social distribution weights as in this report.

Progressivity is defined to mean that the share of the allocation received by low-income States Parties is higher than the share received by higher-income States Parties of the ISA. The reference point could be given by global per capita income of the States Parties. Because S_i is a proportion and $\overline{GNI}$ appears in both the numerator and denominator of the allocation formula for S_i, the reference point of mean or median per capital income of all States Parties.

We assume that the ISA provides what its States Parties view as a fair and just <u>process</u> of deciding upon the allocation formula and the allocation process itself. Given the ISA decision-making process, this assumption can safely be made. Given this assurance of a fair and just process of decision-making, we focus upon fair and equitable outcomes.

Weighting Individual Criteria to Form an Aggregate Index

Individual allocation criteria in the formula $C_i = \prod_{j=1}^{M} C_{ij}{}^{\beta_j}$, whether ordinal or cardinal, must be aggregated in some manner. Index numbers are theoretically consistent formulae that, given some form of weighting, establish an aggregate index (here C_i), where these weights add up to one (herd $\sum_{j=1}^{M} \beta_j = 1$). A method of revealed ethics to derive the weights for the index number's individual criteria (here β_j) is unavailable, in contrast to the social distribution weights (ω_i). As a consequence, each States Party's individual stated ethics must be elicited in some manner.

Individual allocation criteria can be either ordinal or cardinal. Cardinal weights do not present a measurement problem in principle. Individual ordinal criteria that are binary (yes/no) may not pose a special measurement problem, since they are readily converted to a cardinal measure of 1/0, Individual ordinal criteria, however, can present measurement issues if each individual criterion is itself comprised of ordinal rankings. The same type of issue in eliciting States Parties' individual stated ethics follows.

Differences of opinion must be reconciled to arrive at a prioritization that is agreeable to all. This prioritization forms the basis of weights as long as the sum of the weights equals one. This is the opinion aggregation or social choice problem.[27] Some examples of approaches include ranked or cardinal voting systems, the related development of point systems (a form of priority lists), or choice experiment approaches. These could be developed by applying a Delphi approach, perhaps in a web-based method.

Voting Systems. Under voting systems, voters (here individual States Party i) in ranked voting system rank preferences in an ordinal scale.

Borda Voting System If there were, for example, 5 criteria, the top-ranked criteria would then receive a value of 5, the second-ranked criteria would receive a value of 4, etc. This is the Borda process, in which each voter completely ranks all options or candidates and records a score of 0 for the last ranked candidate, 1 for the next-to-last candidate, 2 for next lowest one, and so forth. The total score awarded by all voters determines the winner. The cardinal weight for the top-ranked criteria would then receive a weight of 5/ (5+4+3+2+1). Voters in a cardinal voting system give each candidate an independent rating or grade, say on a scale of 1 to 10 and each criterion then receives a weight of the sum of cardinal ratings by all voters (CPCs of the RFMO) divided by the sum of all cardinal ratings. The potential flaw to the Borda method is that an alternative can be ranked below another even though the first alternative obtains a strict majority over the second alternative. The majority alternative to the Borda method would receive a strict majority of votes when compared pairwise with every other alternative.

Condorcet's Ranking, which addresses the potential flaw to Borda's method, chooses the ranking(s) that are supported by the maximum number of pairwise votes. Condorcet's approach fails, however, if there is not a majority alternative, but does satisfy majority rule for every two adjacent alternatives (the higher ranked alternative has a majority or ties over the lower ranked alternative).

Preferential Voting Systems. Four preferential voting systems for proportional representation, for example, include: The Hare system of single transferable vote, the Borda count, cumulative voting, and additional-remember systems. The Hare system of single transferable vote involves the successive elimination of the lowest-vote candidates, and the transfer of surplus votes of those who have already been elected to other candidates. While Arrow's Impossibility Theorem shows that there is no completely satisfactory method for aggregating individual opinions into a social consensus, aggregation schemes are available that can provide satisfactory answers under almost all conditions. Some form of sealed bid auction is possible, in which CPCs successively bid for their preferred criterion, in which an English style starts from the bottom candidates, a Dutch style starts from the preferred candidates, and there are many options (e.g. choosing the second-best bid). See Brams, S.J. and A. D. Taylor. 1996. *Fair Division —From Cake-Cutting to Dispute Resolution*. Cambridge University Press and Moulon, F. 2003. *Fair Division and Collective Welfare*, M.I.T. Press, Cambridge. Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

Point System. A point system is a type of priority method, and can be developed under certain conditions. Points are awarded to different finite number of attributes based upon some set of criteria. The points could be awarded through voting, in which a voting system results, by an appointed group of experts or representatives of the States Parties, or by a survey of the States Parties. The perceived fairness of the resulting priority formula depends on the legitimacy of the process by which it is established. Differences of opinion must be reconciled in order to arrive at a prioritization that

[27] This discussion draws from Chapter 2 of Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

represents a result perceived as a social consensus and legitimate. Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

A *priority method* based on a given standard distributes the available units to the claimants who have the highest priority.

A *point-allocation procedure* is a procedure under which claimants can allocate a fixed number of points to different goods or issues that reflects, if the claimants are truthful, the importance they attach to receiving these goods or winning on these issues.

If the claimant types are evaluated in a finite number of attributes, and there are a finite number of distinct types of claimants, then a priority method can be represented by a point system if and only if the priority relation is separable. Separability here means the priority relation is separable in attributes 1 and 2 of the priority between t and t' is the same between as the priority between s and s'. If the priority relation is separable in every pair of attributes, it is said to be separable.

Within each dimension, a point system may assign points in a liner or not linear system.
Perceived fairness of a priority formula rests on the legitimacy of the process by which it is determined.

Young, P. 1994. *Equity: How Groups Divide Goods and Burdens Among Their Members*, Princeton University Press, Princeton.

Discrete Choice Experiment A discrete choice experiment is a quantitative technique for eliciting individual preferences. It allows researchers to uncover how individuals value selected attributes of a program, product or service by asking them to state their choice over different hypothetical alternatives. Discrete choice experiments require respondents to state their choice over sets of hypothetical alternatives. Each alternative is described by several characteristics, known as attributes, and responses are used to infer the value placed on each attribute. In comparison to other stated preference techniques that require the individual to rank or rate alternatives, a discrete choice experiment presents a reasonably straightforward task and one which more closely resembles a real-world decision. The method has its theoretical foundation in random utility theory and relies on the assumptions of economic rationality and utility maximization. In stating a preference, the individual is assumed to choose the alternative that yields his/her highest individual benefit, known as utility. Moreover, the utility yielded by an alternative is assumed to depend on the utilities associated with its composing attributes and attribute levels.

Delphi Method: The Delphi method is a forecasting process framework based on the results of multiple rounds of questionnaires sent to States Parties. Several rounds of questionnaires are sent out to the States Parties, and the anonymous responses are aggregated and shared with the group after each round. The States Parties are allowed to adjust their answers in subsequent rounds, based on how they interpret the "group response" that has been provided to them. Since multiple rounds of questions are asked and the panel is told what the group thinks as a whole, the Delphi method seeks to reach the correct response through consensus.

APPENDIX 9. Additional Allocations: Indivisible Goods and Priority Principle[28]

The ISA may decide to use DSM royalties for purposes other than direct distribution to ISA State Parties. Such purposes could entail, for example, projects to mitigate adverse environmental impacts or build scientific capacity through funding scientific research and scientific institutions. These alternative uses represent competing claims for the DSM royalties.

Principles of fairness and equity can also be applied to these other ISA distribution questions. Allocating funds among competing claims or uses, such as projects, can invoke fair and equitable division of the DSM royalties among indivisible, multiple, and heterogeneous claims or uses. This case contrasts with the distribution of ISM royalties among States Parties representing the global population in which all persons have an equal claim upon the homogeneous and perfectly divisible royalties and which inherently can be measured for entitlement through a common metric. The parties no longer have equal claim upon the heterogenous projects or other uses, simple metrics to measure differences in entitlements are no longer available, and Aristotle's Proportionality Principle no longer applies. Instead, the fair division problem revolves around how differences in claims (uses of the DSM royalties) should be evaluated. The projects (claims) are no longer perfectly divisible, and instead are lumpy and indivisible. The uses (claims) are no longer homogeneous, but instead differ and are thereby heterogeneous. The claims are no longer a single use – distribution of DSM royalties to ISA State Parties – but rather multiple uses. A simple metric of funds (US$) allocated to each State Party no longer exists, since each project has its own merit that might be defined and measured in different ways, some of which cannot even be directly measured and quantified by some cardinal measure (but rather by an ordinal measure). (A cardinal number indicates how many of something there are – describes the quantity, such as one, two, or three, and an ordinal number specifies the relative position of something on a list or sequence, such as first, second, or third.)

Decisions in these circumstances can be made by developing lists of objective criteria to make comparative judgments. Each list captures a notion of equity based upon priority rather than the Aristotelian concept of proportionality. Aristotle's Proportionality Principle simply does not apply when a claimant can either receive funding or not (the claims are indivisible). Priority is an ordinal rather than cardinal principle since priority does not indicate the amount by which one deserving claimant is preferred to another – by how much more one claiming is deserving to another. Instead, priority simply indicates that one claimant (use of DSM royalties to fund a project – a claim) is preferred to another, whereas the Proportionality Principle can indicate how much more of the good (DSM royalties) one claimant receives compared to another.

Fairness in the priority case becomes a question of designing a procedure for dividing the DSM royalties among competing indivisible and heterogeneous claims for the royalties that strikes an equitable balance among diverse points of view and that the claimants believe to be visibly fair. Equity principles become the instruments by which States Parties resolve the distributive bargains by establishing a plausible and justifiable basis for the agreement. Equity and fairness then coordinate the expectations of States Parties to establish a plausible basis for agreement. Equitable ways exist (reviewed in Appendix 8) to aggregate individual opinions into a consensus, called the opinion aggregation or social choice problem. Such aggregation occurs through giving weights – relative rankings – to individual criterion to provide an aggregate ranking or score, as discussed in Appendix 8.

[28] This Appendix draws directly from Young (1994).

The concept of equity and fairness in this case becomes the Priority Principle. The Priority Principle requires that allocations among competing claims are made based upon a predetermined ranking of claimants (here us of the DSM royalties among competing projects or uses) – creating a priority list. The claimants are not treated equally, but rather in most situations some claimants will have a stronger a priori claim on the good in question than other claimants do. The relative strengths of a claim depend upon various observable characteristics. This approach establishes priority among competing uses based upon a mixture of prioritized mixture of criteria. Each individual criterion is weighted to give a total score that ranks alternatives. This approach is widely used, for example, to allocate organ transplants such as kidneys. Points systems are one method to score the criteria or attributes and prioritize allocation when there is no complementarity between the attributes that is not captured by the points system.

The priority principle requires two principles of equity, impartiality and consistency, in the criteria used to prioritize claims. An allocation criterion is impartial if the solution depends only upon the description or type of the claimants (projects, uses) in several dimensions or attributes and the total quantity of the good (here funds) to be distributed. For example, a quantity of DSM royalties might be allocated among competing research projects or institutes that can be characterized by some commonly agreed upon dimensions or attributes. Consistency requires that distinctions according to type should be consistently made. An allocation criterion is pairwise consistent if the decision between two claims is always made the same way independently of the other claimants present and how much they receive. The other claimants may affect the number of units the two claimants have to share or to be allocated to them, but it does not determine how the claimants share the amount to be distributed. A standard of comparison is then a list of all types of claimants (projects, uses), ordered from highest to lowest priority.

In sum, priority methods consistently and impartially allocate the DSM royalties over the different indivisible claimants based upon multi-dimensional criteria or attributes that assess each claimant's situation. Other notions of equity, such as utilitarianism or Rawls' difference principle, can contribute in the sense that individual criterion could be based upon the greatest good of utilitarianism or which claimant could benefit the most from an allocation.

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